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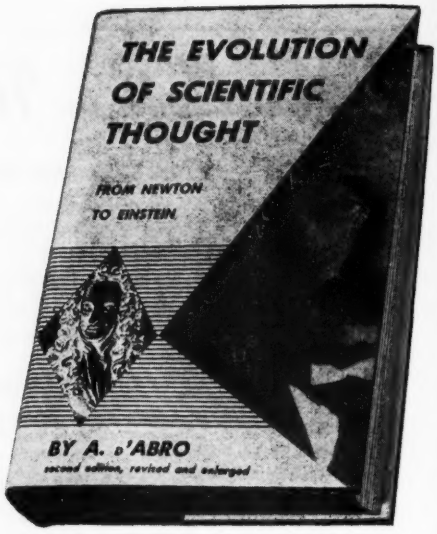

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A CURVE ASSOCIATED WITH A TETRAHEDRON*

VICTOR THÉBAULT, Tennie, France

1. Introduction. The first twelve-point sphere of an orthocentric tetrahedron passes through the extremities of the bimedians and of the bialtitudes. The bimedians meet at the centroid and the bialtitudes at the orthocenter. This suggested the following problem:** Given a tetrahedron $T \equiv ABCD$, determine the locus (Γ) of points M such that the three lines passing through M and meeting respectively two opposite edges of T intersect these edges in six points of a sphere (S).

The object of this article is to report some of the results obtained and to link them to a general theorem concerning pencils of spheres associated with a tetrahedron.

2. Notation. Let O be the center of the circumsphere (O) and G be the centroid of a tetrahedron $T \equiv ABCD$. Let the lengths of the edges of T be denoted by $BC=a$, $DA=a'$, $CA=b$, $DB=b'$, $AB=c$, $DC=C'$.

3. Analytical considerations. In barycentric coordinates, referred to T as the reference tetrahedron, the equations of the curve (Γ) may be written:†

$$
\begin{aligned}
(1) \qquad & a^2yz/(y+z) + a'^2xt/(x+t), \\
& = b^2xz/(x+z) + b'^2yt/(y+t), \\
& = c^2xy/(x+y) + c'^2zt/(z+t),
\end{aligned}
$$

where M has the coordinates (x, y, z, t). The sphere (S) cuts the edges of T in six other points. The lines joining pairs of these points on opposite edges pass through a point M'.‡ The correspondance on (Γ) between the coordinates of M and M' is given by

$$
\begin{aligned}
& a^2yzy'z'/(y+z)(y'+z') = a'^2xtx't'/(x+t)(x'+t'), \\
(2) \qquad & = b^2xzx'z'/(x+z)(x'+z') = b'^2yy't'/(y+t)(y'+t'), \\
& = c^2xyx'y'/(x+y)(x'+y') = c'^2ztz't'/(z+t)(z'+t').
\end{aligned}
$$

The point M whose barycentric coordinates (x, y, z, t) are inversely proportional to the differences

$$
\begin{aligned}
(3) \qquad & b^2 + c^2 + a'^2 - (b'^2 + c'^2 + a^2), \\
& c^2 + a^2 + b'^2 - (c'^2 + a'^2 + b^2), \\
& a^2 + b^2 + c'^2 - (a'^2 + b'^2 + c^2), \\
& a'^2 + b'^2 + c'^2 - (a^2 + b^2 + c^2),
\end{aligned}
$$

---

* Translated from the French by W. E. Byrne, Virginia Military Institute.

** V. Thébault, Intermédiaire des Recherches Mathématiques (I.R.M.), Tome 1, 1945, p. 49.

† A. de Majo, I.R.M., t. 3, 1947, p. 88.

‡ This remark is still valid if sphere (S) is replaced by any quadric. J. Neuberg, Mathesis, 1887, p. 126.

between the sum of the squares of the edges drawn from a vertex and the sum of the squares of the edges of the opposite face, is situated on (Γ). We have for this point:

$$yz/(y+z) = 2(a^2 - a'^2),\ xy/(x+t) = -2(a^2 - a'^2),$$
$$xz/(x+z) = 2(b^2 - b'^2),\ yt/(y+t) = -2(b^2 - b'^2),$$
$$xy/(x+y) = 2(c^2 - c'^2),\ zt/(z+t) = -2(c^2 - c'^2).$$

The barycentric coordinates (x', y', z', t') of the point M' corresponding to M satisfy the relations

$$a^2yzy'z'/(y+z)(y'+z') + a'^2xtx't'/(x+t)(x'+t') = 0, \cdots$$

and

$$a^2/x' + a'^2/y' + a'^2/z' + a^2/t' = 0,$$
$$b'^2/x' + b^2/y' + b'^2/z' + b^2/t' = 0,$$
$$c'^2/x' + c'^2/y' + c^2/z' + c^2/t' = 0,$$

so that

$1/x', 1/y', 1/z', 1/t'$ are proportional to

$$\begin{vmatrix} c'^2 & c^2 & c^2 \\ a'^2 & a'^2 & a^2 \\ b^2 & b'^2 & b^2 \end{vmatrix}, \quad -\begin{vmatrix} c'^2 & c^2 & c^2 \\ a^2 & a'^2 & a^2 \\ b'^2 & b'^2 & b^2 \end{vmatrix},$$

$$\begin{vmatrix} c'^2 & c'^2 & c^2 \\ a^2 & a'^2 & a^2 \\ b'^2 & b^2 & b^2 \end{vmatrix}, \quad -\begin{vmatrix} c'^2 & c'^2 & c^2 \\ a^2 & a'^2 & a'^2 \\ b'^2 & b^2 & b'^2 \end{vmatrix}.$$

In other words the coordinates of M' are inversely proportional to the quantities

$$\begin{aligned} l^2 &= 2(bca')^2 + (b'c'a)^2 - [c'a'b)^2 + (a'b'c)^2 + (abc)^2] \\ m^2 &= 2(cab')^2 + (c'a'b)^2 - [(b'c'a)^2 + (a'b'c)^2 + (abc)^2], \\ n^2 &= 2(abc')^2 + (a'b'c)^2 - [(b'c'a)^2 + (c'a'b)^2 + (abc)^2], \\ p^2 &= 2(a'b'c')^2 + (abc)^2 - [(b'c'a)^2 + (c'a'b)^2 + (a'b'c)^2], \end{aligned} \tag{4}$$

and (1) is satisfied.

4. Geometric considerations. If we consider a sphere (ω), of radius σ, for which the powers of the vertices A, B, C, D of T are kl^2, km^2, kn^2, kp^2, where l^2, m^2, n^2, p^2 are given and k is an arbitrary coefficient, the radical plane (π) of (O) and (ω) satisfies

$$\overline{A\omega}^2 - \sigma^2 = kl^2 = 2O\omega \cdot A\alpha,$$
$$\overline{B\omega}^2 - \sigma^2 = km^2 = 2O\omega \cdot B\beta,$$
$$\overline{C\omega}^2 - \sigma^2 = kn^2 = 2O\omega \cdot C\gamma,$$
$$\overline{D\omega}^2 - \sigma^2 = kp^2 = 2O\omega \cdot D\delta,$$

where α, β, γ, δ are the orthogonal projections of A, B, C, D on (π). When k varies the locus of the center ω of (ω) is the line Δ drawn through O perpendicular to (π); (π) remains fixed and has as its tangential barycentric coordinates l^2, m^2, n^2, p^2.* To two values k_1, k_2 of the parameter k correspond two spheres (ω_1), (ω_2) such that

$$O\omega_1/O\omega_2 = k_1/k_2$$
$$A\alpha + B\beta + C\gamma + D\delta = 4GP$$

where P is the orthogonal projection of the centroid G of T on (π). In the special case where

$$\overline{A\omega}^2 - \overline{B\omega}^2 = k[(a^2 - a'^2) - (b^2 - b'^2)], \quad (5)$$
$$\overline{B\omega}^2 - \overline{C\omega}^2 = k[(b^2 - b'^2) - (c^2 - c'^2)], \cdots$$

Δ coincides with the Euler line OG of T, since

$$\overline{AG}^2 - \overline{BD}^2 = -\tfrac{1}{4}[(a^2 - a'^2) - (b^2 - b'^2)],$$
$$\overline{BG}^2 - \overline{CG}^2 = -\tfrac{1}{4}[(b^2 - b'^2) - (c^2 - c'^2)], \cdots$$

as (π) is then normal to OG, we have $O\omega/OG = -4k$

$$\overline{A\alpha} = -l^2/8\overline{OG}, \quad \overline{B\beta} = -m^2/8\overline{OG}, \quad \overline{C\gamma} = -n^2/8\overline{OG}, \quad \overline{D\delta} = -p^2/8\overline{OG},$$

and

$$(\overline{A\alpha} + \overline{B\beta} + \overline{C\gamma} + \overline{D\delta})/4 = \overline{GP} = -(l^2 + m^2 + n^2 + p^2)/32\overline{OG}. \quad (6)$$

If we write

$$l^2 = b^2 + c^2 + a'^2 - (b'^2 + c'^2 + a^2),$$
$$m^2 = c^2 + a^2 + b'^2 - (c'^2 + a'^2 + b^2),$$
$$n^2 = a^2 + b^2 + c'^2 - (a'^2 + b'^2 + c^2), \quad (7)$$
$$p^2 = a'^2 + b'^2 + c'^2 - (a^2 + b^2 + c^2),$$

we find

$$\overline{A\omega}^2 - \overline{B\omega}^2 = -2[(a^2 - a'^2) - (b^2 - b'^2)], \cdots$$

of the same form as (5). For this choice of l^2, m^2, n^2, p^2 (π) is perpendicular to

* V. Thébault, Mathesis, 1932, Supplément.

OG at a point P such that

$$\overline{GP} = -(l^2 + m^2 + n^2 + p^2)/32\overline{OG} = 0,$$

so P coincides with G. The point M of (Γ) for which the barycentric coordinates are inversely proportional to the quantities (7) is the harmonic pole of the plane (π) perpendicular to OG at G.*

The point M' which corresponds to M is the harmonic pole of the plane (π_1) whose barycentric coordinates are proportional to (4). (π_1) is parallel to the radical plane (π') of (O) and of the sphere determined by

$$l'^2 = (bca')^2 + (b'c'a)^2,$$
$$m'^2 = (cab')^2 + (c'a'b)^2,$$
$$n'^2 = (abc')^2 + (a'b'c)^2,$$
$$p'^2 = (a'b'c')^2 + (abc)^2.$$

If we take values (4) of l^2, m^2, n^2, p^2 we have

$$l^2 - m^2 = 2[(bca')^2 + (b'c'a)^2 - (cab')^2 - (c'a'b)^2] = 2(l'^2 - m'^2), \cdots$$

5. Special tetrahedrons.

A. *The orthocentric tetrahedron.* The centroid G is on (Γ) since (1) is satisfied by $x=y=z=t$. The barycentric coordinates (x', y', z', t') of the point G' which corresponds to G are:

$$1/x' = 2(b^2 + c^2 + a'^2) - (b'^2 + c'^2 + a^2), \cdots$$

G' coincides with the orthocenter H. The sphere (S) is then the first twelve-point sphere of T.

B. *Circumscriptible tetrahedron.* Besides the point M mentioned in section 2 the point M_1 whose barycentric coordinates are inversely proportional to the quantities:

$$l^2 = 2(b + c + a') - (b' + c' + a),$$
$$m^2 = 2(c + a + b') - (c' + a' + b),$$
$$n^2 = 2(a + b + c') - (a' + b' + c),$$
$$p' = 2(a' + b' + c') - (a + b + c),$$

is on (Γ). It is the harmonic pole of the radical plane of (O) and (ω), where l^2, m^2, n^2, p^2 are proportional to the powers of A, B, C, D with respect to (ω). Let G_e be the centroid of the edges of the tetrahedron T. The barycentric coordinates of the point Q symmetric of G_e with respect to G are:

$$a + b' + c', \qquad a' + b + c', \qquad a' + b' + c, \qquad a + b + c.$$

The coordinates of the anticomplimentary point of Q are precisely the values

* A. de Majo and V. Thébault, Mathesis, t, 57, p. 75.

l^2, m^2, n^2, p^2. Furthermore, M_1 coincides with the point of intersection of the lines which join the vertices of the tetrahedron with the Gergonne points of the triangles of the opposite faces. The lines drawn through M_1 meeting opposite edges of the tetrahedron encounter the opposite edges at the points of contact of the sphere tangent to the six edges, so M_1 and M_1' coincide.

NOTE 1. A. de Majo has communicated the following results concerning the harmonic pole M of the plane normal to the Euler line OG at G:

a). *The parallels to the medians of T drawn through M meet the corresponding faces in four coplanar points.*

b). *The six isotomic planes of the planes drawn through M and the edges of T are parallel to the line joining the centroid G to the point Ω of coordinates $b^2+c^2+a'^2$, $c^2+a^2+b'^2$, $a^2+b^2+c'^2$, $a'^2+b'^2+c'^2$.*

NOTE 2. In general, the locus of the harmonic poles of the planes parallel to a fixed plane, with respect to a given tetrahedron T, is a skew cubic which passes through G. If the fixed plane is perpendicular to the Euler line OG, its harmonic pole describes the cubic whose equation in normal coördinates is:

$$\left(b^2 + c^2 + a'^2 + \frac{\lambda}{h_a}\right)x = \left(c^2 + a^2 + b'^2 + \frac{\lambda}{h_b}\right)y$$
$$= \left(a^2 + b^2 + c'^2 + \frac{\lambda}{h_c}\right)z$$
$$= \left(a'^2 + b^2 + c'^2 + \frac{\lambda}{h_d}\right)t,$$

where h_a, h_b, h_c, h_d indicate the lengths of the altitudes AA', BB', CC', DD' of T. This cubic is the isogonal inverse of the line joining the first point of Lemoine K to the point Q_1 of normal coördinates $b^2+c^2+a'^2$, $c^2+a^2+b'^2$, $a^2+b^2+c'^2$, $a'^2+b'^2+c'^2$.

A MINIATURE THEORY IN ILLUSTRATION OF THE CONVOLUTION TRANSFORM*

I. I. HIRSCHMAN, JR. and D. V. WIDDER, Harvard University

1. Introduction. The convolution transform is defined by the equation

$$f(x) = \int_{-\infty}^{+\infty} G(x - t)\phi(t)dt. \tag{1}$$

The kernel $G(x)$ is thought of as given. Equation (1) then "transforms" functions

* Supported in part by the Office of Naval Research under contract NR 045-052.

$\phi(t)$ into functions $f(x)$. If the kernel $G(x)$ is suitably chosen the transform (1) becomes, after an obvious exponential change of variable, the Laplace transform

$$f(x) = \int_0^\infty e^{-xt}\phi(t)dt. \tag{2}$$

For another choice of the kernel, equation (1) becomes

$$f(x) = \int_0^\infty \frac{\phi(t)}{x+t}\,dt, \tag{3}$$

the so-called Stieltjes transform.

The present authors [1]–[4]* have given a fairly complete discussion of the transform for a very general class of kernels. The analytic development of the theory is somewhat involved, but the type of result obtained can be illustrated by use of a special kernel. By use of it we shall obtain directly, without any reference to earlier work, results that are quite analogous to the general ones. They may also serve to illustrate the general theory for the Laplace and Stieltjes transforms, and we shall preface each theorem with a number in braces, { }, referring to the analogous theorems in *The Laplace Transform*,** [6].

The simplification effected by our present choice of kernel is due to the fact that for it equation (1) can be inverted (solved for $\phi(t)$) by a linear differential operator of first order, whereas in general, and in particular for the transforms (2) and (3), the inversion operator must be of infinite order. The results of the present note, though designed primarily as illustrations of a more complex theory, are not without interest in themselves.

2. Notations. Throughout we shall assume that $\phi(t)$ is continuous and that $f(t)$ has a continuous first derivative for all t. More general classes of functions could be considered, but the main points of the theory are brought out by the present simplifying assumptions. We say that $f(t)\in B$ if it is bounded on $-\infty < t < \infty$ and that $f(t)\in L^p$ if

$$\int_{-\infty}^{\infty} |f(t)|^p dt < \infty.$$

Arrows, $\Rightarrow$, $\Leftrightarrow$, are used for implication. The kernel used throughout the remainder of this note is

$$G(t) = 0, \qquad t > 0;$$
$$G(t) = e^t, \qquad t \leqq 0.$$

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* Numbers in brackets refer to the bibliography at the end of this note.

** Roman numbers refer to chapters, arabic to theorems.

3. Convergence.

{VIII 2a} THEOREM 1. *The integral* (1) *converges for some* x_0

$$\Longrightarrow \qquad \textit{the integral converges for all } x.$$

For,

$$\int_{-\infty}^{\infty} G(x-t)\phi(t)dt = \int_{x}^{\infty} e^{x-t}\phi(t)dt.$$

Then

$$\int_{x_0}^{\infty} e^{-t}\phi(t)dt$$

converges by hypothesis, from which the conclusion of the theorem is immediate. As a result of this theorem we may speak of (1) simply as convergent or divergent.

It should be observed that for other kernels the integral (1) may have other convergence properties; {compare II, 1}. The general theory distinguishes between these properties on the basis of characteristics of the kernel.

4. Inversion.

{VII 6a, VIII 9} THEOREM 2. (1) *converges*

$$\Longrightarrow \qquad f(x) - f'(x) = \phi(x), \qquad -\infty < x < \infty.$$

That is, the inversion operator, with our present choice of kernel, is the differential operator $(1-D)$, where D indicates differentiation with respect to x. The proof is immediate. We have

$$f(x) = e^{x}\int_{x}^{\infty} e^{-t}\phi(t)dt,$$

$$f'(x) = e^{x}\int_{x}^{\infty} e^{-t}\phi(t)dt - \phi(x), \qquad -\infty < x < \infty.$$

The result now follows by subtracting these two equations.

5. Representation. We wish now to investigate what classes of functions $f(x)$ can be represented in the form (1).

{VII 11a, VIII 13} THEOREM 3.

$$f(x) \sim Ae^{x}, \qquad (\textit{some constant } A,\ x \to +\infty),$$

$$\Longrightarrow \qquad f(x) = \int_{\infty}^{\infty} G(x-t)[f(t) - f'(t)]dt + Ae^{x}, \qquad -\infty < x < \infty$$

For, we have after integration by parts,

$$\int_x^R e^{x-t}f'(t)dt = f(R)e^{x-R} - f(x) + \int_x^R e^{x-t}f(t)dt,$$

(4)

$$\int_x^R e^{x-t}[f(t) - f'(t)]dt = f(x) - f(R)e^{x-R}.$$

Let $R\to\infty$. By hypothesis the right hand side tends to $f(x)-Ae^x$ so that

$$\int_x^\infty e^{x-t}[f(t) - f'(t)]dt = f(x) - Ae^x. \tag{5}$$

This equation is equivalent to the conclusion of the theorem.

COROLLARY 3.1 $\lim_{x\to+\infty} e^{-x}f(x)=0$

$$\Longrightarrow \quad f(x) = \int_{-\infty}^\infty G(x-t)[f(t) - f'(t)]dt.$$

COROLLARY 3.2 $\int_0^\infty e^{-t}[f(t)-f'(t)]dt$ *converges*

$$\Longrightarrow \quad f(x) \sim Ae^x, \qquad x\to+\infty,$$

where

$$A = f(0) - \int_0^\infty e^{-t}[f(t) - f'(t)]dt. \tag{6}$$

This conclusion follows from equation (4), where now it is the left-hand side which is known to approach a limit as $R\to+\infty$. Equation (6) follows from (4) when $x=0$, $R=+\infty$. Thus we see that equation (5) is valid whenever the integral converges.

6. Bounded $\phi(t)$. We now characterize the functions $f(x)$ which have the representation (1) with bounded $\phi(t)$.

{VII 16a, VIII20a} THEOREM 4. $f(x), f'(x)\in B$

$$\Longleftrightarrow \quad f(x) = \int_{-\infty}^\infty G(x-t)\phi(t)dt,$$

where $\phi(t)\in B$.

We observe first that $G(t)\geqq 0$ for all t and that

$$\int_{-\infty}^\infty G(x-t)dt = \int_{-\infty}^0 e^t dt = 1.$$

First assume the representation (1) with $|\phi(t)| \leqq M$ for all t. Then

$$|f(x)| \leqq \int_{-\infty}^{\infty} G(x-t)|\phi(t)|\,dt \leqq M\int_{-\infty}^{\infty} G(x-t)dt = M$$

$$|f(x) - f'(x)| = |\phi(x)| \leqq M,$$

from which it follows that $f(x), f'(x) \in B$.

Conversely, from the boundedness of $f(x)$ it is clear that

$$\lim_{x\to\infty} e^{-x}f(x) = 0.$$

By Corollary 3.1

$$f(x) = \int_{-\infty}^{\infty} G(x-t)\phi(t)dt,$$

where $\phi(t) = f(t) - f'(t)$. Since f and f' are bounded, the same is true of ϕ.

COROLLARY 4. $f(x) - f'(x) \in B$

$$\iff \quad f(x) = \int_{-\infty}^{\infty} G(x-t)\phi(t)dt + Ae^{x},$$

where $\phi(t) \in B$ and A is some constant.

Half of this conclusion follows from Theorem 2, the other half from Corollary 3.2.

7. Positive $\phi(t)$. Here we investigate the analogue of Bernstein's classic representation of completely monotonic functions.

{VII 14a, VIII 17c} THEOREM 5.

$$f(x) \geqq 0, \qquad f(x) - f'(x) \geqq 0, \qquad (-\infty < x < \infty),$$

$$\iff \quad f(x) = \int_{\infty}^{\infty} G(x-t)\phi(t)dt + Ae^{x},$$

where $\phi(t) \geqq 0$ and $A \geqq 0$.

If $f(x)$ has the convolution representation described in the theorem one sees by inspection that $f(x)$ and $f(x) - f'(x) = \phi(x)$ are non-negative functions.

Conversely, we have from equation (4) that

$$\int_0^R e^{-t}[f(t) - f'(t)]dt = f(0) - f(R)e^{-R}.$$

Since we are now assuming that $f(t) \geqq 0$ and $f(t) - f'(t) \geqq 0$, we have

$$\int_0^R e^{-t}[f(t) - f'(t)]dr \leqq f(0),$$

$$\int_0^\infty e^{-t}[f(t) - f'(t)]dt \leqq f(0). \tag{7}$$

Hence we may apply Corollary 3.2 to obtain

$$f(x) = \int_{-\infty}^{\infty} G(x - t)\phi(t)dt + Ae^x,$$

where $\phi(t) = f(t) - f'(t) \geqq 0$. From equation (6) and inequality (7) we see that $A \geqq 0$, and our result is proved.

In the light of Corollary 4 we might seek to alter the present theorem in such a way as to eliminate the hypothesis $f(x) \geqq 0$. That it is not redundant for the present theorem is evident from the function $f(x) = -xe^x$. Since it changes sign it cannot have the representation of Theorem 5. Yet $f(x) - f'(x) = e^x > 0$. However, we have the following alternative result.

COROLLARY 5.1

$$\begin{aligned} &1.\ f(x) - f'(x) \geqq 0, && -\infty < x < \infty, \\ &2.\ \lim_{x \to \infty} e^{-x}f(x) = 0 \end{aligned}$$

$$\Longleftrightarrow \qquad f(x) = \int_{-\infty}^{\infty} G(x - t)\phi(t)dt,$$

where $\phi(t) \geqq 0$.

This follows immediately from Theorem 2 and Corollary 3.1

COROLLARY 5.2

$$\begin{aligned} &1.\ f(x) - f'(x) \geqq 0, && -\infty < x < \infty, \\ &2.\ \lim_{x \to +\infty} e^{-x}f(x) = 0 \end{aligned}$$

$$\Longrightarrow \qquad f(x) \geqq 0, \qquad -\infty < x < \infty.$$

This is a consequence of Corollary 5.1. It is of interest, independent as it is of any representation theory.

8. Functions $\phi(t)$ in L^p. In general theory these classes must be discussed separately according as $p > 1$ or $p = 1$. Here the two cases coalesce into a single one.

{VII 15a, VIII 19a} THEOREM 6. $f(x), f'(x) \in L^p$, $(p \geqq 1)$,

$$\Longleftrightarrow \qquad f(x) = \int_{-\infty}^{\infty} G(x - t)\phi(t)dt,$$

where $\phi(t) \in L^p$.

First suppose that $f(x)$ has the integral representation described. Then if $p=1$

$$|f(x)| \leqq \int_{-\infty}^{\infty} G(x-t)|\phi(t)|\,dt,$$

and by Fubini's theorem*

$$\begin{aligned}\int_{-\infty}^{\infty}|f(x)|\,dx &\leqq \int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}G(x-t)|\phi(t)|\,dt\\ &\leqq \int_{-\infty}^{\infty}|\phi(t)|\,dt\int_{-\infty}^{\infty}G(x-t)dx\\ &= \int_{-\infty}^{\infty}|\phi(t)|\,dt.\end{aligned}$$

Thus $f(x)\in L$. Since $f(x)-f'(x)=\phi(x)\in L$, it is clear that $f'(x)\in L$.

If $p>1$, we have by use of Hölder's inequality

$$f(x) = \int_{-\infty}^{\infty}[G(x-t)]^{1/2}[G(x-t)]^{1/2}\phi(t)dt,$$

$$|f(x)| \leqq \left[\int_{-\infty}^{\infty}[G(x-t)]^{q/2}dt\right]^{1/q}\left[\int_{-\infty}^{\infty}[G(x-t)]^{p/2}|\phi(t)|^p dt\right]^{1/p},$$

where $1/p+1/q=1$.

Evaluating the first integral on the right and again using Fubini's theorem, we obtain

$$\begin{aligned}\int_{-\infty}^{\infty}|f(x)|^p dx &\leqq \left[\frac{2}{q}\right]^{p/q}\int_{-\infty}^{\infty}|\phi(t)|^p dt\int_{-\infty}^{\infty}[G(x-t)]^{p/2}dx\\ &\leqq \frac{2}{p}\left[\frac{2}{q}\right]^{p/q}\int_{-\infty}^{\infty}|\phi(t)|^p dt.\end{aligned}$$

Thus it is again evident, from Minkowski's inequality,** that f and $f'\in L^p$.

Conversely, if $f-f'\in L^p$ it is clear from Hölder's inequality** (or from the boundedness of e^{-t} if $p=1$) that

$$\int_0^{\infty} e^{-t}[f(t)-f'(t)]dt$$

converges. Hence we may apply Corollary 3.2 to obtain

$$f(x) = \int_{\infty}^{\infty} G(x-t)\phi(t)dt + Ae^x,$$

* See, for example, E. C. Titchmarsh, The Theory of Functions, Oxford 1932, p. 392.

** See E. C. Titchmarsh, loc. cit., pp. 383–384.

where $\phi(t)=f(t)-f'(t)\in L^p$. Then the function defined by this integral $\in L^p$ by the first half of the proof. Since $e^x\notin L^p$ and $f(x)\in L^p$ we see from Minkowski's inequality (or directly if $p=1$) that $A=0$. This concludes the proof.

COROLLARY 6. $f(x)-f'(x)\in L^p$ $(p\geqq 1)$

$$\Longleftrightarrow \qquad f(x)=\int_{-\infty}^{\infty} G(x-t)\phi(t)dt + Ae^x,$$

where $\phi(t)\in L^p$ and A is some constant.

This follows from Theorem 2 and Corollary 3.2

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MATHEMATICAL NOTES

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NOTE ON MARTIN'S ERGODIC FUNCTION

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1. **Introduction.** Let M be a fixed bounded point set in the plane containing more than one point; let ϵ denote a positive number. If C is a continuous, rectifiable curve in the plane such that each point of M is within distance ϵ of some point of C, then M. H. Martin (*Ergodic curves*, American Journal of Mathematics, vol. 58 (1936); *The ergodic function*, Bulletin of the American Mathematical Society, vol. 43 (1937)) calls C *ϵ-ergodic* to M. Let $\rho>0$ be the radius of the smallest circle containing M; for a given ϵ let $\Lambda(\epsilon)$ be the greatest lower

bound of the lengths of the continuous, rectifiable curves ϵ-*ergodic* to M. Martin has shown that $\Lambda(\epsilon)=0$ if and only if $\epsilon \geqq \rho$, and that, for all positive ϵ, the function $\Lambda(\epsilon)$ is continuous, monotone non-increasing and non-negative. He proves that there is a continuous, rectifiable curve ϵ-ergodic to M and having length $\Lambda(\epsilon)$.

To show that $\Lambda(\epsilon)$ is finite for each ϵ, he exhibits a family of spirals ϵ-ergodic to M and of finite length. In this note we remark that $\Lambda(\epsilon)$ is strictly monotone decreasing and bounded above by a function more simple than the lengths of the spirals.

2. **Theorem.** We shall establish the following result.

THEOREM 1. *The function* $\Lambda(\epsilon)$ *is strictly monotone decreasing on* $0<\epsilon<\rho$. *In fact, if* $0<a<b<\rho$, *then* $\Lambda(a)-\Lambda(b) \geqq b-a>0$.

Proof. Let

$$C: \quad x = x(t), \qquad y = y(t), \qquad 0 \leqq t \leqq 1,$$

be an a-ergodic curve to M with length $\Lambda(a)$. We first observe that $\Lambda(a)>b-a$. If this were not true, an arbitrary point P of M could be joined to $(x(0), y(0))$ by a path of length not exceeding b by first joining P to C by a path of length not greater than a and then joining this point to the initial end-point of C by a path along C of length not greater than $b-a$. Thus, since $b<\rho$, the inequality $\Lambda(a) \leqq b-a$ is impossible.

Let t^* be the value of the parameter t such that the part C^* of C corresponding to $0 \leqq t \leqq t^*$ has length $b-a$. We shall now show that $C-C^*$, a non-degenerate, proper sub-curve of C given by $t^* \leqq t \leqq 1$, is b-ergodic to M. If P is a point of M within distance a of $C-C^*$, then P is within distance b of $C-C^*$ since $b>a$. If P is farther than a from $C-C^*$, then its distance to some point Q of C^* is not greater than a. Since Q is within distance $b-a$ of $(x(t^*), y(t^*))$, the distance of P from this end-point of $C-C^*$ is not more than b. Thus $\Lambda(b)$ is smaller than $\Lambda(a)$ by at least the positive number $b-a$.

3. **A further result.** We shall establish another inequality, as expressed in the following theorem.

THEOREM 2. *For all* $0<\epsilon \leqq \rho$ *we have*

$$\rho - \epsilon \leqq \Lambda(\epsilon) < \pi\rho\left(\frac{\rho}{\epsilon} + 3\right).$$

Proof. The inequality on the left is an immediate consequence of the inequality in Theorem 1 and $\Lambda(\rho)=0$.

Let S denote the circle of radius ρ bounding M. For a given ϵ let n be the positive integer for which $n-1<\rho/\epsilon \leqq n$. We shall construct a continuous, rectifiable curve from circles concentric with S and held together by a radius of S. As our system of circles we take the $n-1$ circles concentric with S and having

radii $\rho - j\epsilon$, $j = 1, 2, \cdots, n-1$. Any point of the interior of S (hence any point of M) is within distance ϵ of a circle of the system. Clearly the number of circles could be decreased if $n > 3$. The sum of the lengths of circles of the system is

$$2\pi\left((n-1)\rho - \frac{n(n-1)}{2}\epsilon\right).$$

If we add a radius of S, we secure a continuous, rectifiable curve having length less than $\Lambda(\epsilon)$. In particular, we have

$$\Lambda(\epsilon) < 2\pi\left(n\rho - \frac{n(n-1)}{2}\epsilon\right).$$

Since $n \geqq \rho/\epsilon$ and $n < \rho/\epsilon + 1$ we have

$$\Lambda(\epsilon) < \pi\rho\left(\frac{\rho}{\epsilon} + 3\right).$$

4. Discussion. It would be of interest to know what the shortest ergodic curves are in case M consists of a finite number of points. In case M consists of only two points, the shortest ergodic curve, for a fixed ϵ, is a straight line segment. In case M consists of three points, the shortest ergodic curve is easily proved to be a straight line segment or two straight line segments with a common end-point. The two cases arise from considering the various possible positions of the three points (cf. Courant and Robbins, *What Is Mathematics*, Oxford, 1941, p. 336 et seq. for a lemma useful in determining the position of the curve). The question asked here could be phrased as follows: A paper boy is to throw papers at $n > 1$ given houses in a plane. If he can throw a paper ϵ feet and can have his papers delivered to him at any point in the plane, what route has a minimum length?

As is seen in the simplest case (when M consists of two points), $\Lambda = \Lambda(\epsilon)$, $0 < \epsilon \leqq \rho$, has a linear graph in the (ϵ, Λ)-plane. Professor Martin has conjectured (in a letter to the author) that for any point set M, if $\Lambda(\epsilon)$ is not linear, it is convex. That is, for any two values ϵ_1, ϵ_2 selected from the interval $0 < \epsilon \leqq \rho$ we have

$$\Lambda(\lambda\epsilon_1 + \mu\epsilon_2) \leqq \lambda\Lambda(\epsilon_1) + \mu\Lambda(\epsilon_2),$$

where $\lambda \geqq 0$, $\mu \geqq 0$ and $\lambda + \mu = 1$.

In addition to these questions one might ask when the (generally improper) Riemann integral of $\Lambda(\epsilon)$ exists on $0 \leqq \epsilon \leqq \rho$. What does the linearity of $\Lambda(\epsilon)$ on an interval of $0 < \epsilon \leqq \rho$ imply concerning the structure of M? When is $\Lambda(\epsilon)$ differentiable?

EUCLIDEAN n-SPACE ANALOGUE OF A THEOREM OF GALILEO

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1. **Introduction.** The purpose of this paper is to establish an n-space analogue of the following theorem due* to Galileo: *The area of a circle is the mean proportional between the areas of any two similar polygons, of which one is circumscribed about the circle, and the other is isoperimetric with the circle.*

2. **Theorem.** We shall establish the following result.

THEOREM: *Let V_1 denote the n-dimensional content of a sphere in Euclidean n-space, V the n-dimensional content of a circumscribed polytope, and v that of a similar polytope whose $(n-1)$-dimensional content is the same as that of the sphere. Then $Vv^{n-1}=V_1^n$.*

3. **Proof.** Let r denote the radius of the sphere, and S_1, S, and s respective $(n-1)$-dimensional contents, so that $s=S_1$ by hypothesis. Since the circumscribed polytope is composed of pyramids of altutide r, having a common apex at the center of the sphere, we have $V=Sr/n$, and similarly $V_1=S_1r/n$. Since the two polytopes are similar, it follows that

$$\left(\frac{V}{v}\right)^{1/n}=\left(\frac{S}{s}\right)^{1/n-1}.$$

Solving for v, we get $v=V(s/S)^{n/n-1}$. Therefore we have

$$Vv^{n-1}=\left(\frac{Sr}{n}\right)\left(\frac{Sr}{n}\right)^{n-1}\left(\frac{S_1}{S}\right)^n=\left(\frac{S_1r}{n}\right)^n=V_1^n.$$

4. **Conclusion.** Hence we have a direct generalization† of Galileo's theorem: *The volume of a sphere in Euclidean n-space is the geometric mean of the volume of a circumscribed polytope, and that of a similar polytope whose $(n-1)$-dimensional content is the same as that of the sphere, with weights 1 and $n-1$, respectively.*

A PROPERTY OF MINKOWSKIAN CIRCLES

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1. **Introduction.** Let a Euclidean circle C with center A be given. Move C rigidly to circle C_0 with center A_0 on C, and let A_1 be one of the intersection points of C and C_0. If now C_0 is moved to circle C_1 with center at A_1, then C and C_1 intersect at A_0 and at a second point A_2. Again, if C_1 is moved to C_2 with center at A_2, then C and C_2 intersect at A_2 and at a new point A_3. The fact that

* The writer found the above theorem entitled *Galileo's Theorem* as Ex. 135, pp. 377, in "Plane and Solid Geometry," 1899, by William J. Milne.

† The writer is greatly indebted to Professors E. F. Beckenbach and H. S. M. Coxeter for their aid in the preparation of this paper.

in this process the new circle always passes through a previous intersection point can be called the "re-entrant" property of a circle with respect to its center. The intersection sequence, of course, contains only six points, or, the circle "goes around itself six times."

2. **Theorem.** If the motions above are restricted to translations, the process can be carried out, with identical results, on Minkowskian circles. Or, with translations understood, the same thing can be stated for the Euclidean plane as follows.

THEOREM 1. *Every simple, closed, strictly convex curve which has a center is re-entrant with respect to that center and goes around itself six times.*

3. **Proof.** Let such a curve C with center A be given, and let A_0 be an arbitrary point on C. The line t determined by A and A_0 cuts C again at A_3. Take P a variable point on C and consider the chord which has P as an end-point and whose line is parallel to t. It is well known that the segment $\overline{A_0A_3}$ has the unique maximal length for all such chords, and that as P moves away from A_0 the corresponding chord length decreases monotonically until P reaches a position where its chord line is a supporting line of C and the chord length is zero. As P moves away from A_0, on one side of t, let A_1 be the first point at which the chord length is $A_0A_3/2$, and let the second end of this chord be A_2. Let the line of A_1 and A cut C in A_4, and the line of A_2 and A cut C in A_5. Join these points determined on C in the index order to form an inscribed hexagon. By the center property of C and the equality of vertical angles, it follows from congruent triangles that the opposite sides of this hexagon are equal in length and on parallel lines. Moreover, opposite sides have length half that of the chord determined by the remaining two vertices, and this third chord is parallel to the opposite sides. To show this, take T a translation which moves C to C' so that A falls, for example, on A_4, and let P be an intersection point of C and C'. Then P and $T^{-1}(P)$ determine a chord of C whose line is parallel to that of A_1 and A_4, and whose length is $A_1A_4/2$. Since C has only two such chords, namely $\overline{A_0A_5}$ and $\overline{A_2A_3}$, it follows from the position of the vertices that P is A_3 or A_5. If A is moved to any vertex of the hexagon, then, C in its new position passes through the adjacent vertices. Starting with A_0, which was arbitrary, then, C has the re-entrant property and the intersection sequence closes with A_5.

4. **Another result.** If a simple, closed, convex curve has the re-entrant property, with respect to an interior point A, and if P and Q are two consecutive points in an intersection sequence, it is easy to see that the chord through A, which is parallel to the line determined by P and Q, must be bisected by A. By the use of continuity this can be extended to give the following result.

THEOREM 2. *If a simple, closed, strictly convex curve C has the re-entrant property with respect to an interior point A, then A is the center of C.*

EQUATIONS OF COMPATIBILITY FOR FINITE DEFORMATION OF A CONTINUOUS MEDIUM*

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This note contains a concise derivation, in explicit form, of the integrability conditions for the case of finite deformations. These do not appear to be contained in the existing literature on the theory of finite deformations.**

If a continuous medium, occupying a region of three-dimensional Euclidean space, is subjected to a one-to-one continuous deformation such that the points of a line element of length ds_0 are displaced to the points of a line element of length ds, then the Eulerian strain tensor e_{ij} is defined by

$$(ds)^2 - (ds_0)^2 = 2e_{ij}dx^i dx^j, \qquad (i, j = 1, 2, 3), \tag{1}$$

where x^1, x^2, x^3 are the coordinates of a point in the deformed space. These coordinates are, in general, curvilinear. Using the same set of coordinates to designate points of the medium before and after deformation, we define the covariant metric tensors g_{ij} and h_{ij} of the deformed and undeformed spaces, respectively, by the fundamental quadratic forms:

$$\begin{cases} (ds)^2 = g_{ij}dx^i dx^j, \\ (ds_0)^2 = h_{ij}dx^i dx^j. \end{cases} \tag{2}$$

Inasmuch as the manifold in question is Euclidean, the Riemann tensor R_{ijkl}, based on either g_{ij} or h_{ij}, must vanish. If this condition is applied to the tensor h_{ij}, by using g_{ij} as the fundamental metric of the space in which the e_{ij} are defined, one obtains a set of six non-linear partial differential equations, called the *equations of compatibility*.

In the following derivation, covariant differentiation is performed with respect to the metric tensor g_{ij}, and a prime is used to distinguish Riemann tensors and Christoffel symbols based on h_{ij} from those based on g_{ij}.

Comparing (1) and (2) we get

$$h_{ij} = g_{ij} - 2e_{ij}, \tag{3}$$

so that

$$h_{ij,k} = \frac{\partial h_{ij}}{\partial x^k} - \left\{ \begin{matrix} \alpha \\ ik \end{matrix} \right\} h_{j\alpha} - \left\{ \begin{matrix} \alpha \\ jk \end{matrix} \right\} h_{i\alpha} = -2e_{ij,k}. \tag{4}$$

A calculation of Christoffel's symbols $[ij, k]'$ yields

* I am indebted for the suggestion of the method of deducing these equations to Professor I. S. Sokolnikoff.

** A paper by V. Z. Vlasov, entitled "The Equations of Continuity of Deformations in Curvilinear Coordinates," Journal of Applied Mathematics and Mechanics, vol. 8 (1944), pp. 301–306, utilizes ideas somewhat related to those of this note. However, in deriving his equations, Vlasov used variational techniques which restricted him to the consideration of infinitesimal deformations.

$$[ij, k]' = \left\{ \begin{matrix} \alpha \\ ij \end{matrix} \right\} h_{k\alpha} - e_{ijk}, \tag{5}$$

where e_{ijk} is defined by

$$e_{ijk} \equiv e_{ik,j} + e_{kj,i} - e_{ij,k}. \tag{6}$$

Since contravariant metric tensors g^{ij} and h^{ij} are

$$g^{ij} \equiv G^{ij}/g, \qquad h^{ij} \equiv H^{ij}/h, \tag{7}$$

where G^{ij} and H^{ij} are the cofactors of g_{ij} and h_{ij} in $|g_{ij}| = g$ and $|h_{ij}| = h$, respectively, it follows that

$$\left\{ \begin{matrix} \beta \\ ij \end{matrix} \right\}' = h^{k\beta}[ij, k]' = \left\{ \begin{matrix} \beta \\ ij \end{matrix} \right\} - h^{k\beta} e_{ijk}. \tag{8}$$

In order to obtain the Riemann tensor R'_{ijkl}, we compute

$$\begin{aligned} \frac{\partial}{\partial x^k}[jl, i]' &= \left(\frac{\partial}{\partial x^k} \left\{ \begin{matrix} \alpha \\ jl \end{matrix} \right\} \right) h_{\alpha i} + \left\{ \begin{matrix} \alpha \\ jl \end{matrix} \right\} \frac{\partial h_{\alpha i}}{\partial x^k} - \frac{\partial e_{jli}}{\partial x^k} \\ &= \left(\frac{\partial}{\partial x^k} \left\{ \begin{matrix} \beta \\ jl \end{matrix} \right\} + \left\{ \begin{matrix} \alpha \\ jl \end{matrix} \right\} \left\{ \begin{matrix} \beta \\ \alpha k \end{matrix} \right\} \right) h_{\beta i} + \left\{ \begin{matrix} \alpha \\ jl \end{matrix} \right\} \left\{ \begin{matrix} \beta \\ ik \end{matrix} \right\} h_{\alpha\beta} \\ &\quad + \left\{ \begin{matrix} \alpha \\ jl \end{matrix} \right\} h_{\alpha i,k} - e_{jli,k} - \left\{ \begin{matrix} \alpha \\ jk \end{matrix} \right\} e_{\alpha li} - \left\{ \begin{matrix} \alpha \\ lk \end{matrix} \right\} e_{j\alpha i} - \left\{ \begin{matrix} \alpha \\ ik \end{matrix} \right\} e_{jl\alpha}. \end{aligned}$$

Also, from (5) and (8) we have

$$\begin{aligned} \left\{ \begin{matrix} \alpha \\ jk \end{matrix} \right\}' [il, \alpha]' &= \left\{ \begin{matrix} \alpha \\ jk \end{matrix} \right\} \left\{ \begin{matrix} \beta \\ il \end{matrix} \right\} h_{\alpha\beta} - \left\{ \begin{matrix} \alpha \\ il \end{matrix} \right\} e_{jk\alpha} \\ &\quad - \left\{ \begin{matrix} \alpha \\ jk \end{matrix} \right\} e_{il\alpha} + h^{\alpha\beta} e_{jk\beta} e_{il\alpha}, \end{aligned}$$

and from (4) and (6), we deduce that

$$h_{\alpha i,k} = -2e_{\alpha i,k} = -(e_{ik\alpha} + e_{\alpha ki}).$$

Therefore

$$\begin{aligned} \frac{\partial}{\partial x^k}[jl, i]' + \left\{ \begin{matrix} \alpha \\ jk \end{matrix} \right\}' [il, \alpha]' &= \left(\frac{\partial}{\partial x^k} \left\{ \begin{matrix} \beta \\ jl \end{matrix} \right\} + \left\{ \begin{matrix} \alpha \\ jl \end{matrix} \right\} \left\{ \begin{matrix} \beta \\ \alpha k \end{matrix} \right\} \right) h_{\beta i} \\ &\quad + A_{ijkl} - e_{jli,k} + h^{\alpha\beta} e_{jk\beta} e_{jl\alpha}, \end{aligned}$$

where

$$A_{ijkl} \equiv \left(\left\{ \begin{matrix} \alpha \\ jl \end{matrix} \right\} \left\{ \begin{matrix} \beta \\ ik \end{matrix} \right\} + \left\{ \begin{matrix} \alpha \\ ik \end{matrix} \right\} \left\{ \begin{matrix} \beta \\ jl \end{matrix} \right\} \right) h_{\alpha\beta} + \left\{ \begin{matrix} \alpha \\ jl \end{matrix} \right\} h_{\alpha i,k} + \left\{ \begin{matrix} \alpha \\ jk \end{matrix} \right\} h_{\alpha i,l} - \left\{ \begin{matrix} \alpha \\ lk \end{matrix} \right\} e_{j\alpha i} - \left\{ \begin{matrix} \alpha \\ ik \end{matrix} \right\} e_{jl\alpha} - \left\{ \begin{matrix} \alpha \\ il \end{matrix} \right\} e_{jk\alpha}.$$

Clearly, the system A_{ijkl} is symmetric with respect to the third and fourth indices. If we impose the condition that the Riemann tensor R'_{ijkl} is a zero tensor, we get

$$R'_{ijkl} = \frac{\partial}{\partial x^k} [jl, i]' - \frac{\partial}{\partial x^l} [jk, i]' + \left\{ \begin{matrix} \alpha \\ jk \end{matrix} \right\}' [il, \alpha]' - \left\{ \begin{matrix} \alpha \\ jl \end{matrix} \right\}' [ik, \alpha]' = 0;$$

and the use of the foregoing formulas yields

$$R'_{ijkl} = h_{\beta i} R^{\beta}_{jkl} + e_{ijkl} + h^{\alpha\beta}(e_{jk\beta}e_{il\alpha} - e_{jl\beta}e_{ik\alpha}),$$

since, by (6),

$$e_{jki,l} - e_{jli,k} = e_{ik,jl} + e_{jl,ik} - e_{il,jk} - e_{jk,li} \equiv e_{ijkl}.$$

However, we have $R^{\beta}_{jkl} = 0$, and hence the equations satisfied by the strain tensor are:

$$e_{ijkl} + h^{\alpha\beta}(e_{jk\beta}e_{il\alpha} - e_{jl\beta}e_{ik\alpha}) = 0.$$

If the second order terms are neglected, these expressions reduce to the usual form for the equations of compatibility,

$$e_{ijkl} \equiv e_{ik,jl} + e_{jl,ik} - e_{il,jk} - e_{jk,li} = 0,$$

familiar in the theory of infinitesimal deformations.

SOME ELEMENTARY IDENTITIES OF ORDERED ABELIAN SETS

A. L. Foster, University of California, Berkeley

1. Introduction. By slightly generalizing and abstracting the relations between exponents which occur in the representation of the endomorphisms of an Abelian group as a (pseudo) matrix ring, one finds as a by-product a number of elementary ordering identities which seem to be of some independent interest. It is the purpose of this note to call attention to several of these, which are faintly reminiscent of four point ordering theorems of projective geometry (Theorem 5).

2. Results. An ordered Abelian set $S=(S, <, +)$ is an ordered set $(S, <)$

(*i*) $$\left.\begin{array}{l} a < b,\ a = b, \text{ or } b < a \text{ (exactly one)} \\ a < b \quad \text{and} \quad b < c \quad \text{imply } a < c \end{array}\right\} a, b, c \in S,$$

such that S is a subset of an Abelian group $(G, +)$,

(*ii*) $$S \subseteq G,$$

and where

(*iii*) $\qquad a, b \in S \quad \text{and} \quad a \leqq b \qquad \text{imply } b - a \in S.$

In an ordered Abelian set $(S, <, +)$, $(S, +)$ need not be closed with respect to $+$, nor need $<$ be defined throughout G. From (*iii*), however, one has

(2.1) $\qquad 0 \in S.$

In the special case where $S = G$ (="ordered Abelian group"), (*iii*) is evidently redundant; an ordered Abelian group is then simply an Abelian group $(G, +)$ whose elements have been ordered in any way (*i*).

For example, for given m, the set of all integers $0 \leqq x \leqq m$ is an ordered Abelian set with respect to ordinary $+$, and $<$ =natural order. Again the subset 0, 3, 6, 9 of the additive group of whole numbers is an ordered Abelian set if $<$ refer to the natural order, but not if $<$ is determined by

$$0 < 6 < 3 < 9.$$

Again, for example, the natural numbers 0, $\pm 3, 6, 9, 12, \cdots$, ordered by

$$0 < -3 < 3 < 9 < 6 < 12 < 15 < 18 < \cdots,$$

form an ordered Abelian set as do also 0, $\pm 3, -6, -9, -12, \cdots$, ordered by

$$0 < 3 < -3 < -6 < -9 < -12 < \cdots.$$

Among the elementary functions definable in an ordered Abelian set $S = (S, <, +)$, for all a, b, c of S, we consider

(2.2) $\qquad |a, b| = \begin{cases} b - a & \text{if } a \leqq b, \\ a - b & \text{if } b \leqq a; \end{cases}$

(2.3) $\qquad /a, b/ = \begin{cases} b - a & \text{if } a \leqq b, \\ 0 & \text{if } b < a; \end{cases}$

(2.4) $\quad [a, b, c] = b'$, where $a' \leqq b' \leqq c'$, and $a', b', c' = a, b, c$, in some order.

It is seen that $[a, b, c]$ is uniquely defined by (2.4), that each of the three functions (2.2)–(2.4) has S as its range, and that (2.2) and (2.4) are symmetric in all arguments,

(2.5) $\qquad \begin{aligned} |a, b| &= |b, a|, \\ [a, b, c] &= [a, c, b] = [b, a, c] = \textit{etc.} \end{aligned}$

Relative to a given ordered Abelian set $(S, <, +)$ one may call $|a, b|$ the "absolute range" between a and b, $/a, b/$ the "forward range" from a to b, and $[a, b, c]$, the "middle" of the set a, b, c.

THEOREM 1. *In any ordered Abelian set we have*

(2.6) $\qquad /a, b/ + /b, c/ - /a, c/ = |b, [b, c, a]|.$

The proof of (2.6) is seen by direct verification for each of the possible relative orderings of a, b, c. If

$$
\begin{aligned}
&a \leqq b \leqq c \quad \text{both sides of (2.6) reduce to} \quad 0,\\
&a \leqq c \leqq b \quad \text{both sides of (2.6) reduce to} \quad b - c,\\
&b \leqq a \leqq c \quad \text{both sides of (2.6) reduce to} \quad a - b,\\
&b \leqq c \leqq a \quad \text{both sides of (2.6) reduce to} \quad c - b,\\
&c \leqq a \leqq b \quad \text{both sides of (2.6) reduce to} \quad b - a,\\
&c \leqq b \leqq a \quad \text{both sides of (2.6) reduce to} \quad 0.
\end{aligned} \tag{2.7}
$$

In view of the fact that S is in general not closed with respect to $+$ (or with respect to $-$), the following corollary is of interest:

COROLLARY. *In an ordered Abelian set,* $/a, b/ + /b, c/ - /a, c/$ *is an element of* S *for all* a, b, c *in* S.

As in Theorem 1, we have the following result:

THEOREM 2. *In any ordered Abelian set we have*

$$|x, y| + |y, z| - |x, z| = 2|y, [y, z, x]|. \tag{2.8}$$

(*Here* $2t = t + t$).

If one abbreviates

$$
\begin{aligned}
/xyz \cdots st/ &= /x, y/ + /y, z/ + \cdots + /s, t/\\
|xyz \cdots st| &= |x, y| + |y, z| + \cdots + |s, t|,
\end{aligned} \tag{2.9}
$$

by induction one obtains the following results:

THEOREM 3. *In any ordered Abelian set we have*

$$
\begin{aligned}
/xyz \cdots uvw/ = /x, w/ &+ |y, [x, y, z]| + |z, [z, t, x]|\\
&+ \cdots + |u, [u, v, x]| + |v, [v, w, x]|.
\end{aligned} \tag{2.10}
$$

THEOREM 4. *In any ordered Abelian set we have*

$$
\begin{aligned}
|xyz \cdots uvw| = |x, w| &+ 2|y, [y, z, x]| + 2|z, [z, t, x]|\\
&+ \cdots + 2|u, [u, v, x]| + 2|v, [v, w, x]|.
\end{aligned} \tag{2.11}
$$

Again by direct verification of the six possible arrangements of a, b, c one proves:

THEOREM 5. *In an ordered Abelian set we have*

$$|a, [a, b, c]| + |b, [b, c, d]| = |b, [b, a, d]| + |a, [a, d, c]|. \tag{2.12}$$

If the left of (2.12) is denoted by $\Pi(a, b; c, d)$ then (2.12) may be written

$$\Pi(a, b; c, d) = \Pi(b, a; d, c).$$

CLASSROOM NOTES

EDITED BY C. B. ALLENDOERFER, Haverford College,

All material for this department should be sent to C. B. Allendoerfer, Haverford College, Haverford, Pa.

MATHEMATICS "AU NATUREL"

N. A. COURT, University of Oklahoma

In a recent article of this MONTHLY (Artificial Mathematics, vol. 57, No. 5, May, 1950, pp. 329–330) Professor F. H. Miller takes sharp issue with writers of textbooks, say, in differential equations or the calculus who present their solutions and proofs in an arbitrary, *deus ex machina* fashion, without letting the reader in on the secret which led to those solutions.

The transgressing authors are indicted on three counts: (1) Such a procedure on the part of the said authors tends to undermine the faith of college teachers of mathematics that the science they profess is based on logic and that the solutions they offer are arrived at in a rational way. (2) Those artificial ways have a discouraging effect upon the students who begin to doubt their ability to do mathematical thinking of their own if such "crystal ball methods" are an integral part of it. (3) Such unmotivated mathematical arguments are unnecessary and unjustified. They can readily be avoided if a little care and thought is devoted to the situation.

To take the last point first, there is very little doubt that the solutions objected to could be dressed up so that they would look "natural" and would appeal to our sense of "reasonableness." But if that neat appearance of theirs is meant to suggest that this was the path followed by their discoverers, it would be sadly misleading. Mathematical truths, solutions of mathematical problems, are seldom arrived at in such a matter-of-fact fashion. The proofs offered for them by their discoverers themselves have in most cases very little to do with the form in which these results were first conceived. The great masters of mathematical thought have been quite outspoken about this state of affairs (see Jacques Hadamard, *An Essay on the Psychology of Invention in the Mathematical Field*, Princeton University Press, 1945). Is being hit on the head by an apple falling off a tree a strictly logical approach to the invention of the theory of gravitation?

But there is no need to climb to such dizzy heights, or even to go as high as differential equations, to discover "artificial" elements in mathematical proofs. Almost any proof in plane geometry offers convincing evidence of it. Take the theorem of the sum of the angles of a triangle (Euclid, Book I, prop. 32). After a line has been drawn through the vertex of the triangle parallel to the opposite side, the proof is very simple and very "natural." But "where on earth does that parallel line come from?" Of all the things one might do, how does one know that the parallel line is the thing which will do the job?

We might as well tell our students that solving problems involves an element of inventiveness and that they will have to supply it if they are to be successful in their mathematical endeavors. Perhaps by the time they are ready to study differential equations it is not too early to shock them with this disturbing and challenging piece of news.

And while we are divulging secrets, we might perhaps let our students in on another one we have been guarding from them with zeal worthy of a better cause. In the problems we propose to our students we are very careful to supply all the data relevant to their solution, and we are just as careful to keep out anything that is irrelevant thereto. We might as well tell them that this is artificial, that it is done in order to make their work easier. In problems they may actually be confronted with that may not be the case. In fact, the most difficult part of their task may be precisely the separation of what is relevant from what is not. We might even go so far as to offer them problems where such a discrimination would have to be resorted to. Perhaps it would be advisable to start on this course much earlier. Suppose we amuse ourselves by asking a seventh grader to work a problem like the following.

One end of a string 4.5 ft. long is attached to an end of a diagonal of a rectangular white piece of paper 1/16 of an inch thick; the two sides of the rectangle passing through that point are 3 in. and 4 in. long. The other end of that same string is fixed to the point of intersection of the diagonals of a rectangular ceiling painted green, in a room that is 8 ft. high. Find the length of the diagonal of the paper and of the diagonal of the ceiling.

So much about the students. As to the college teachers of mathematics themselves, they can find in their subject so many solid virtues with which to bolster up their professional enthusiasm that they can, without much regret, dispense with one of questionable validity.

THE GENERAL SINE AND COSINE CURVES

E. L. EAGLE, University of Tennessee

In many textbooks on analytic geometry, and trigonometry, the equations of the general sine and cosine curves are given expression, canonically, in the form:

$$y = k \sin (bx + c)$$
$$y = k \cos (bx + c).$$

However, there is an advantage to writing such equations in the following form:

$$y = a \sin n(x - \alpha)$$
$$y = a \cos n(x - \alpha).$$

Then, a will be the amplitude and $2\pi/n$ the period (as usual), but the lag or lead will be indicated, definitely, by the point $(\alpha, 0)$. For example: Given

$y=2/3 \sin (3x+\pi/3)$, we convert to canonical form:

$$y = \frac{2}{3} \sin 3\left(x - \left[-\frac{\pi}{9}\right]\right).$$

Therefore, the amplitude is $2/3$ and the period is $2\pi/3$; and, to sketch the curve, we may conveniently start a cycle at $(-\pi/9, 0)$.

This approach removes any uncertainty, when the curve is out of phase, as to whether we have a lag or a lead.

It is gratifying to note that the taking of α from behind a minus sign is a procedure familiar to the student for he has already done the same thing in dealing with the circle, parabola, ellipse, and hyperbola.

ELEMENTARY PROBLEMS AND SOLUTIONS

EDITED BY HOWARD EVES, Oregon State College

Send all communications concerning Elementary Problems and Solutions to Howard Eves, Mathematics Department, Oregon State College, Corvallis, Oregon. This department welcomes problems believed to be new, and demanding no tools beyond those ordinarily furnished in the first two years of college mathematics. To facilitate their consideration, solutions should be submitted on separate, signed sheets, within three months after publication of problems.

PROBLEMS FOR SOLUTION

E 941. *Proposed by P. A. Piza, San Juan, Puerto Rico*

Find positive integers a, b, c such that $a^2+b^2+3c^2=(a+b+c)^2$.

E 942. *Proposed by L. A. Ringenberg, Eastern Illinois State College*

In the sequence of powers of 2: 2, 4, 8, 16, 32, $\cdots$, the units digits form a sequence of period four. What periodic properties do the other digits have?

E 943. *Proposed by S. H. Gould, Purdue University*

By analogy with the motion of the planets, it seems natural to assume for any central motion that, at least if the particle never passes through the center, (a) the distance of the particle from the center is a maximum only when the velocity of the particle is a minimum, (b) the velocity is a maximum only when the distance is a minimum. Prove (a) and give a counterexample of (b).

E 944. *Proposed by E. R. Bowersox, Chicago, Ill.*

A truncated cone with one end open is to be formed from a circular sheet, cutting out a central portion for the base, and using the remaining annular ring for the curved area. Solve for the truncated cone of maximum volume.

E 945. *Proposed by Leo Moser, University of North Carolina*

If all the faces of a convex polyhedron have central symmetry show that there are at least eight vertices where exactly three edges meet. (The cube has exactly eight such vertices.)

SOLUTIONS

Odd and Even Derangements

E 907 [1950, 184]. *Proposed by C. D. Olds, San Jose State College*

A derangement of n ordered letters is a permutation of the n letters which leaves no letter fixed. Find formulas which will give the number of odd and the number of even derangements of n ordered letters.

Solution by S. T. Thompson, Tacoma, Washington. Let O_n, E_n, and D_n denote the numbers of odd, even, and all derangements of the n letters. One easily finds $O_1=E_1=0$, $O_2=1$, $E_2=0$. We now determine a recursion formula for O_n, $n>2$. Starting with the original arrangement of the n letters, transpose the first and kth letters. There will then be E_{n-2} derangements of the remaining $n-2$ letters which lead to odd derangements of the entire set. Since k may be any one of the integers $2, 3, \cdots, n$, there are $(n-1)E_{n-2}$ odd derangements of this type. Again, starting with the original arrangement of the n letters, consider the first letter followed by any one of the E_{n-1} even derangements of the last $n-1$ letters. Now transpose the letters in the first and kth positions, and we obtain an odd derangement of all n letters. There are $(n-1)E_{n-1}$ odd derangements of this type. The two types of odd derangements considered are mutually exclusive and exhaustive. Therefore

$$O_n = (n-1)(E_{n-1} + E_{n-2}), \qquad n > 2.$$

Similarly

$$E_n = (n-1)(O_{n-1} + O_{n-2}), \qquad n > 2.$$

From these results one now easily establishes for all positive n, by mathematical induction, that

$$O_n - E_n = (-)^n(n-1). \tag{1}$$

But it is known that

$$O_n + E_n = D_n = n!\sum_{i=0}^{n}(-)^i/i!.$$

Therefore

$$O_n = \{D_n + (-)^n(n-1)\}/2,$$
$$E_n = \{D_n - (-)^n(n-1)\}/2.$$

Also solved by D. H. Browne, C. F. Pinzka, James Singer, G. W. Walker, and the proposer.

The Proposer obtained the relation (1) by considering the determinant $\Delta(n)$ of order n all of whose elements are equal to 1, except those of the main diagonal, which equal 0. The value of the determinant is $(-)^{n-1}(n-1)$. Now let $j_1, j_2, \cdots, j_n$ be any permutation of the integers $1, 2, \cdots, n$. Then we can expand $\Delta(n)$ in the form

$$\Delta(n) = \sum \pm a_{1j_1}a_{2j_2} \cdots a_{nj_n}.$$

Now any term in this sum which does not correspond to a derangement $j_1, j_2, \cdots, j_n$ of the n integers $1, 2, \cdots, n$ contains an element from the main diagonal and so is zero. The value of a term which does correspond to a derangement will be $+1$ or -1 according as the permutation is even or odd. Thus the value of $\Delta(n)$ is $E_n - O_n$, and we obtain relation (1).

A Property of 1/2

E 908 [1950, 184]. *Proposed by R. J. Koch, Tulane University*

Show that $r=1/2$ is the only rational value of r for which the geometric series $\sum ar^n$, $a \neq 0$, has the property that there exists a term equal to the sum of all terms from some point on, and that when $r=1/2$ this property holds for every term.

Solution by Walter Fleming, Mankato State Teachers College. The sum after m terms, for $-1<r<1$, is $ar^m/(1-r)$. Equating this to ar^n we have

$$r^m/(1-r) = r^n.$$

For $-1<r<0$ it is necessary that $m<n$ and we can write

$$r^{n-m+1} - r^{n-m} + 1 = 0.$$

This equation has no rational roots. For $0<r<1$ it is necessary that $m>n$ and we can write

$$r^{m-n} + r - 1 = 0.$$

This equation has no rational root except if $m-n=1$, in which case $r=1/2$ is a root.

It is clear that for $r=1/2$, $r^{n+1}/(1-r)=r^n$, for all n, which means that each term is equal to the sum of all the terms after it.

In the trivial case, $r=0$, the required property holds for every term except the first.

Also solved by R. E. Crane, Ragnar Dybvik, David Ellis, J. E. Freund, Roger Lessard, P. Na Nagara, C. S. Ogilvy, W. O. Pennell, Alex Rosenberg, Azriel Rosenfeld, C. M. Sandwick, O. D. Smith, Charles Solky, O. E. Stanaitis, Alfred Sylvester, C. W. Trigg, G. W. Walker, J. W. Young, and the proposer. Many of these solutions were incomplete.

Limit of a Determinant

E 910 [1950, 184]. *Proposed by R. M. Gordon, Carnegie Institute of Technology*

If $k>0, f_i(x)$ are independent, $f_i^{(n)}(x)$ exist and are well behaved, find

$$\lim_{h\to 0} h^{-k} \begin{vmatrix} f_1(x) & f_2(x) & \cdots f_n(x) \\ f_1(x+h) & f_2(x+h) & \cdots f_n(x+h) \\ f_1(x+2h) & f_2(x+2h) & \cdots f_n(x+2h) \\ \cdot & \cdot & \cdots \quad \cdot \\ f_1[x+(n-1)h] & f_2[x+(n-1)h] & \cdots f_n[x+(n-1)h] \end{vmatrix}.$$

Solution by O. E. Stanaitis, St. Olaf College, Minn. By adding to each row multiples of the rows above it as suggested by the difference operator

$$\Delta^m f(x) = \sum_{j=0}^{m} (-1)^j \binom{m}{j} f[x+(m-j)h]$$

we may write the given expression in the form

$$\lim h^{(n-1)n/2-k} \begin{vmatrix} f_1(x) & f_2(x) & \cdots & f_n(x) \\ \dfrac{\Delta f_1(x)}{h} & \dfrac{\Delta f_2(x)}{h} & \cdots & \dfrac{\Delta f_n(x)}{h} \\ \dfrac{\Delta^2 f_1(x)}{h^2} & \dfrac{\Delta^2 f_2(x)}{h^2} & \cdots & \dfrac{\Delta^2 f_n(x)}{h^2} \\ \cdot & \cdot & \cdots & \cdot \\ \dfrac{\Delta^{n-1} f_1(x)}{h^{n-1}} & \dfrac{\Delta^{n-1} f_2(x)}{h^{n-1}} & \cdots & \dfrac{\Delta^{n-1} f_n(x)}{h^{n-1}} \end{vmatrix}.$$

The limit of the determinant is the well known Wronskian, which, according to the conditions under consideration, is finite and different from zero. It follows that the limit of the expression is the Wronskian for $k=(n-1)n/2$, equals zero for $k<(n-1)n/2$, and does not exist for $k>(n-1)n/2$.

Also solved by Kurt Eisemann, Roger Lessard, Norman Miller, and the proposer.

Miller stated that for $k=(n-1)n/2$, the theorem in question is a special case of a more general theorem in which $x, x+h, x+2h, \cdots, x+(n-1)h$ are replaced by $x_1, x_2, \cdots, x_n$ and in which the denominator h^k is replaced by a Vandermonde determinant. This more general theorem, again, arises from a generalized theorem of mean value of which the case $n=3$ is given in Goursat-Hedrick, *Mathematical Analysis*, vol. 1, p. 10. References to the more general theorems and proofs are as follows: H. A. Schwarz, *Collected Works*, vol. 2, p. 296; T. J. Stieltjes, *Collected Works*, p. 105; N. Miller, *Trans. of the Royal Soc. of Canada*, 1931, p. 195.

Miller also mentioned, as an allied problem, that of showing that the determinant

$$H = \begin{vmatrix} 1 & x & x^2 & \cdots & x^n \\ 1 & x+h & (x+h)^2 & \cdots & (x+h)^n \\ 1 & x+2h & (x+2h)^2 & \cdots & (x+2h)^n \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 1 & x+nh & (x+nh)^2 & \cdots & (x+nh)^n \end{vmatrix}$$

is independent of x. This may be done by showing that $dH/dx \equiv 0$. The value of the determinant, found by taking $x=0$, is

$$1!\,2!\,3!\cdots n!\,h^{n(n+1)/2}.$$

The special case, $k=3$, $n=3$, of the given problem, is found in Widder, *Advanced Calculus*, ex. 13, p. 220.

Professor Umbugio's Expedition

E 911 [1950, 259]. *Proposed by G. B. Huff, University of Georgia*

The following item appeared in the April 1, 1913, issue of the Erewhon Daily Howler: "Science lovers of Guayazuela will be disappointed to learn that the antarctic expedition of Professor Euclide Paracelso Bombasto Umbugio, which was planned to start the first of next month and last through November, has been temporarily postponed. In making his characteristically thorough preparations, the eminent numerologist discovered that June 13 of this year falls on Friday and has decided that he will postpone the trip until a year when no Friday the thirteenth occurs in the seven months of the expedition. The Professor, an authority on the doctrine of probability, assured the press that the delay will be of short duration. He explained that the probability of an auspicious month is 6/7 and that hence the probability of seven consecutive auspicious months is $(6/7)^7$ or a little more than 2/5. From this the genius infers that the probability of postponing the trip next year is less than 3/5 and the probability that it will be postponed for nine consecutive years is $(3/5)^9$ or less than 1/100."

Of course, we now know that the Professor has never made his trip, though four times nine years have passed. Is the Professor having a tremendous run of bad luck?

Solution by C. F. Pinzka, Xavier University, Cincinnati. Beginning with May 13 of any year, the days elapsing until May 13, June 13, $\cdots$, November 13 are congruent modulo 7 to 0, 3, 5, 1, 4, 6, 2, respectively. Since these numbers form a complete residue system, one of these dates must fall on a Friday.

Or, a glance at any calendar will show that the 13th day of May to November, inclusive, is always on a different day of the week, whence one of these dates must always fall on a Friday.

Also solved by Martin Berman, A. D. Bradley, G. M. Dillon, Ragnar Dybvik, Emil Grosswald, E. S. Keeping, J. R. F. Kent, Roger Lessard, Marie A. Moore, C. S. Ogilvy, Bart Park, Azriel Rosenfeld, R. M. Simons, G. W. Walker, A. Wilansky, and R. V. Andree's freshman mathematics class at the University of Oklahoma.

The Professor's probability argument falls down, of course, because calendar months are not independent of each other. It is interesting that the thirteenth of a month is more likely to be Friday than any other day of the week. See E 36 [1933, 607].

ADVANCED PROBLEMS AND SOLUTIONS

EDITED BY E. P. STARKE, Rutgers University

Send all communications concerning Advanced Problems and Solutions to E. P. Starke, Rutgers University, New Brunswick, New Jersey. All manuscripts should be typewritten, with double spacing and margins at least one inch wide. Problems containing results believed to be new or extensions of old results are especially sought. Proposers of problems should also enclose any solutions or information that will assist the editor. In general, problems in well known textbooks or results in readily accessible sources should not be proposed for this department.

PROBLEMS FOR SOLUTION

4415. *Proposed by R. P. Boas, Jr., and W. K. Hayman, Brown University*

Find all the values of α and β for which the series

$$\sum_{n=1}^{\infty} n^{\alpha} \sin (n^{\beta})$$

converges.

4416. *Proposed by D. J. Newman, New York University*

Let $a_1, a_2, a_3, \cdots$ be a sequence of integers such that $\sum_{n=1}^{\infty} 1/a_n$ diverges. Show that almost all integers have a factor in common with some a_n.

4417. *Proposed by Victor Thébault, Tennie, Sarthe, France*

(1) The necessary and sufficient condition that an orthocentric tetrahedron $T \equiv ABCD$ can be bordered by isosceles tetrahedrons $T_a \equiv A_1BCD$, $T_b \equiv B_1CDA$, $T_c \equiv C_1DAB$, $T_d \equiv D_1ABC$ is that all the face angles of T be acute. (2) The tetrahedrons T and $T_1 \equiv A_1B_1C_1D_1$ are orthologic. (3) The lines AA_1, BB_1, CC_1, DD_1 are four hyperbolic lines. (4) The sum of the squares of the areas of the faces of T_a, T_b, T_c, T_d is equal to the sum of the squares of the products of the opposite edges of T.

4418. *Proposed by A. C. Aitken, Edinburgh, Scotland*

Evaluate:

$$\begin{vmatrix} \frac{1}{1!} & -1 & \cdots & \cdots & \cdots \\ \frac{2^2}{2!} & \frac{1}{1!} & -2 & \cdots & \cdots \\ \frac{3^3}{3!} & \frac{2^2}{2!} & \frac{1}{1!} & -3 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{n^n}{n!} & \frac{(n-1)^{n-1}}{(n-1)!} & \frac{(n-2)^{n-2}}{(n-2)!} & \cdots & \frac{1}{1!} \end{vmatrix}.$$

4419. *Proposed by L. G. Jones, University of Oregon, and Alex Rosenberg, University of Toronto.*

In a paper, Remarks on Divisors of Zero, this MONTHLY [1942, 286–295], N. H. McCoy stated and proved the following theorem: Let $f(x)$ be an element of $R[x]$, where R is a commutative ring. Then $f(x)$ is a divisor of zero in $R[x]$ if and only if there exists a nonzero element c of R such that $cf(x)=0$.

Show that, if R is not assumed to be commutative, the theorem no longer holds.

SOLUTIONS

Area Having Lowest Centroid

4336 [1949, 186]. *Proposed by Orrin Frink, Pennsylvania State College*

Find the arc of fixed length $l>2a$, lying below the x-axis and joining the points $(-a, 0)$ and $(a, 0)$, which includes between itself and the x-axis an area of lowest possible center of gravity. This will be the form actually assumed by a weightless flexible arc supported at its ends if it is holding water.

Solution by Frank Herlihy, Comstock, Michigan. Let:

R = tension in the cable, which will be constant throughout its length since it is itself massless and the liquid cannot exert any shear on it;
ρ = radius of curvature of the cable;
σ = mass per unit area of the liquid;
g = gravitational constant;
and let y be measured downward from the x-axis.

Then the pressure at depth y below the axis is σgy. If the cable is in equilibrium the pressure σgy and the radius ρ at the point at distance y below the x-axis will determine $R=-\sigma gy\rho$ or

$$\sigma g y \frac{(1 + |\, dy/dx\,|^2)^{3/2}}{d^2y/dx^2} = -R.$$

With $R/\sigma g = a^2/4$, an integration gives

$$y^2 = \tfrac{1}{2}a^2(1 + |\, dy/dx\,|^2)^{-1/2} + c = h^2 - a^2 \sin^2 \tfrac{1}{2}\theta,$$

where $\theta = \arctan(dy/dx)$, and $h = (c + \frac{1}{2}a^2)^{1/2}$ is the maximum value of y.

In order that y shall vanish at the supports, $a^2 > h^2$. Let $h^2 = k^2a^2$ where $k^2 < 1$, and let $k \sin\phi = -\sin\frac{1}{2}\theta$. Then $y = h\cos\phi$ and since

$$\sin\theta = dy/ds = -h(\sin\phi)(d\phi/ds)$$

we shall have

$$\frac{ds}{d\phi} = -h\frac{\sin\phi}{\sin\theta} = \frac{a}{2}\frac{1}{\sqrt{1 - k^2\sin^2\phi}}.$$

Thus

$$s = \frac{a}{2}\int_0^{\phi} \frac{d\phi}{\sqrt{1 - k^2\sin^2\phi}} = \tfrac{1}{2}aF(k, \phi).$$

Similarly

$$\frac{dx}{d\phi} = -h\frac{\sin\phi}{\sin\theta}\cos\theta = \frac{a}{2}\frac{1 - 2k^2\sin^2\phi}{\sqrt{1 - k^2\sin^2\phi}},$$

$$x = a\int_0^{\phi} \sqrt{1 - k^2\sin^2\phi}\, d\phi - \frac{a}{2}\int_0^{\phi} \frac{d\phi}{\sqrt{1 - k^2\sin^2\phi}}$$

$$= aE(k, \phi) - \tfrac{1}{2}aF(k, \phi).$$

If α is the half span of the problem, then

$$\alpha = aE(k, \tfrac{1}{2}\pi) - \tfrac{1}{2}aF(k, \tfrac{1}{2}\pi)$$

since $y = h\cos\phi$ implies $\phi = \frac{1}{2}\pi$ when $y = 0$. Also $\frac{1}{2}l = \frac{1}{2}aF(k, \frac{1}{2}\pi)$.

If k and a are fixed, the curve is given in the parameter ϕ by

$$y = h\cos\phi, \qquad x = aE(k, \phi) - \tfrac{1}{2}aF(k, \phi).$$

This solution is merely Euler's solution of the Elastica, identical except for the values of k and a. See MacMillan, *Statics and Dynamics of a Particle*, pp. 196–200, where also curves are drawn for various values of k (including both the case where there are inflections, which is the present problem, and the non-inflectional case where $k^2 \geqq 1$).

RECENT PUBLICATIONS

EDITED BY E. P. VANCE, Oberlin College

All books for review should be sent directly to the editor of this department, American Mathematical Monthly, 531 West 116th Street, New York 27, N. Y. and not to any of the other editors or officers of the Association.

A Short Course in Differential Equations. By E. D. Rainville. New York, The Macmillan Company, 1949. 9+210 pages.

Upon reflection on the objective of the book which is being reviewed, one concludes that it was designed as a text for students who need a more complete set of recipes for the solution of the commonly occuring ordinary differential equations than he would find in the usual calculus book. The student reader, however, is not expected to have reached the stage where the term "theorem" belongs in a mathematics text. When a result is stated for the reader, he frequently finds a footnote indicating other hypotheses or the location of a precise statement and proof.

The standard material of a first differential equations course is included with the exception that neither power nor Fourier series solutions are included nor is any treatment given of systems of equations. The highlight of the presentation is the inclusion of a large number of examples in each section, so that the student may see in rather complete detail the procedure suitable for the particular equation being treated. However, one may question the advisability of the inclusion of several examples with no variation in technique whatsoever in some sections.

The book is well supplied with exercises and answers are included for virtually all the problems. The space devoted to applications is given to a more detailed study of the problems of the vibrating string and the electric circuit rather than a more superficial treatment of numerous applications.

PAUL CIVIN

An Introduction to College Geometry. By E. H. Taylor and G. C. Bartoo. New York, The Macmillan Company, 1949. 6+143 pages. 112 figures. $3.15.

The book is a revision of one which appeared in 1939, and of which the reviewer could find no mention in this MONTHLY. It is designed for prospective teachers of secondary mathematics who have had no synthetic geometry since high school; also to stimulate students interested in geometry to study the discoveries made in this subject since Euclid.

No knowledge of mathematics is presupposed save that from the usual course of plane geometry in high school; no attempt is made to stress the logical foundations of the subject, nor any use made of imaginary elements in algebra. However, on page 108, the cross ratio of a pencil of lines is defined in terms of the sines of the angles. Historical notes are scattered throughout the text wherever appropriate.

The first four chapters deal with directed lines and angles, points at infinity, similar figures and loci. Chapter five is devoted to the properties of the triangle,

but nothing more modern than the Theorems of Ceva, Pascal and Desargues, the nine-point circle and the Simson line. The remaining chapters are concerned with harmonic ranges and pencils, pole and polar as to a circle, orthogonal and coaxal circles, cross ratio and the principle of duality.

Numerous exercises of various degrees of difficulty appear in each chapter. The appendix lists 108 theorems from elementary plane geometry for ready reference in the text and 22 constructions the reader should know. Here may be found 70 additional exercises and 50 construction problems if the student desires supplementary work.

Within the scope of the book it is well written and should serve as a good introduction to a second course in the modern geometry of the triangle and circle. The press-work and the proof-reading seem excellent. One slip occurs on page 59 where Menelaus is credited as living around 100 B.C. instead of 100 A.D.

J. R. MUSSELMAN

NEW BOOKS RECEIVED

From Euclid to Eddington, the Tarner Lectures, 1947. By Sir E. Whittaker. Cambridge University Press, 1949. 10+212 pp.

Analysis and Design of Experiments. By H. B. Mann. New York, Dover, 1949. 10+198 pp. $2.95.

La Geometrie Contemporaine. ("Que sais-je?," no. 401.) By A. Delachet. Paris, Presses Universitaires de France, 1950. 128 pp.

An Index of Nomograms. By D. P. Adams. New York, Wiley, 1950. 10+174 pp. $4.00.

College Algebra. By H. A. Bender. New York, Pitman, 1950. 12+451 pp. $3.50.

Leçons sur la Théorie des Fonctions. Fourth Edition. By E. Borel. Paris, Gauthier-Villars, 1950. 12+295 pp.

Experimental Designs. By W. Cochran and G. Cox. New York, Wiley, 1950. 10+454 pp. $5.75.

Monografia sobre los Números. By F. J. Duarte. Caracas, Tipografia Americana, 1949. 246 pp.

Premiers principes de géométrie moderne. Third Edition. By E. Duporq. Paris, Gauthier-Villars, 1949. 173 pp.

Plane Trigonometry. By Gordon Fuller. New York, McGraw-Hill, 1950. 13+270 pp. $2.75.

Éléments de Physique Moderne Théorique. By Georges Guinier. Paris, Bordas, 1949. 159 pp.

Gelöste und Ungelöste Mathematische Probleme aus Alter und Neuer Zeit. By Heinrich Tietze. Biederstein Publishers, Munich, 1949. Volume I: xx+256 pp.; Volume II: iv+305 pp. Paper Bound 18 DM, Linen Bound 25 DM (German Marks).

Science is a Sacred Cow. By Anthony Standen. New York, E. P. Dutton and Company, 1950. 221 pp. $2.75.

CLUBS AND ALLIED ACTIVITIES

EDITED BY L. F. OLLMANN, Hofstra College

Send reports of all activities, such as club reports, special features, topics with references, student papers, and other material of interest to L. F. Ollmann, Hofstra College, Hempstead, New York.

CLUB REPORTS, 1949–50

Newtonian Society, Lehigh University

The *Newtonian Society* of Lehigh University is composed of sophomores and juniors who have demonstrated outstanding ability in mathematics during their freshman year.

During the academic year 1949–50, the society devoted several meetings to reorganization. The following informal talks were presented at the remaining meetings:

Some applications of number theory, by Prof. G. W. Petrie III

Stellar astronomy, by Prof. R. N. Van Arnam

Numerical analysis, by Prof. V. W. Latshaw

Homogeneous functions and homogeneous differential equations, by Prof. J. W. Mettler.

In February the constitution was changed to provide for the election of officers in February instead of in September. It was also decided to obtain certificates of membership for all members.

The present officers are: President, J. Burk McNamara; Vice-President and Treasurer, Frederick Kieshauer; Secretary, Alan Koppes; Faculty Advisors, Prof. J. O. Chellevold, Prof. G. W. Petrie III (fall semester).

Kappa Mu Epsilon, Hofstra College

The *New York Alpha* Chapter of *Kappa Mu Epsilon* held regular monthly meetings at which the following talks were presented:

Mathematics in the schools of England, by Geoffrey Charlesworth

The four-color problem, by Prof. L. F. Ollmann

Verb tense determined symbolically, by William Beller

Complex roots of polynomials determined graphically, by Prof. H. F. Fehr of Columbia Teachers College

Non-Euclidean geometry, by Perry Watts

Lissajous's figures, by Otto Juelich

Biometry, by Jane Brandt.

In addition to the regular meetings, the chapter held a Christmas party and square dance, a picnic and an initiation banquet at which Prof. J. E. Eaton of Queens College spoke on *The nature of applied mathematics.*

John Richard Prussen was the winner of the award offered to the best student of first year mathematics.

The chapter was represented by five faculty and five student members at the *Kappa Mu Epsilon* luncheon held in New York City at the time of the Christ-

mas meeting of the M.A.A.

The national officers of *Kappa Mu Epsilon* met on the Hofstra campus during the Christmas holidays.

During the past semester, the chapter sponsored student help sections. *Kappa Mu Epsilon* members gave their time to help those who came at the assigned periods.

Prof. L. F. Ollmann, chairman of the department, and national treasurer of *Kappa Mu Epsilon*, installed the *Pennsylvania Alpha* Chapter of *Kappa Mu Epsilon* at Westminster College, New Wilmington, Pennsylvania on May 17th.

The officers for 1950–51 are: President, Peter Hinrichs; Vice-President, Richard Jaeger; Secretary, Emily Schuppel; Treasurer, Peter Marshall; Historian, Jane Brandt; Corresponding Secretary, Lysle Marshall; Faculty Sponsor E. Marie Hove.

Mathematics Club, Case Institute of Technology

The activities of the *Mathematics Club* of the Case Institute of Technology for the year 1949–50 were entirely those of lectures by both students and members of the faculty. These lectures were:

The method of least squares, by Dr. F. C. Leone

The problem of trisecting an angle, by Irwin Wieder

The Königsberg bridge problem, by Dr. C. Saltzer

Conformal mapping, by George Millburn.

The new officers are: President, James T. Morse; Vice-President, Ernst Prasse.

Pi Mu Epsilon, Bucknell University

The papers presented to the *Pennsylvania Beta* Chapter of *Pi Mu Epsilon* for the academic year 1949–50 were:

Binomial theorem, by James Bortner

Group theory, by Donald Ohl

Practical equation solution, by Edwin Irland

Applications of the finite calculus, by Evan Boden.

Other activities included a business meeting, a social, and a picnic. Average attendance at each meeting was 30.

The officers for 1950–51 are: Director, John W. Sprout; Vice-Director, F. R. Stauffer; Secretary, Charlotte Stratton; Treasurer, R. E. Jones, Jr.

Kappa Mu Epsilon, Alabama College

The *Alabama Gamma* Chapter held regular meetings from September through May. In October a party was given for all students in the college taking mathematics. In March nine new members were initiated and a banquet followed the initiation ceremony.

The following papers were presented at other meetings:

Number is everything, by Joyce Moncrief

The number system, by Annie Falls.

Officers elected for the year 1950–51 are: President, Joyce Moncrief; Vice-President, Betty Ellis; Secretary, Gay Penn; Treasurer, Annie Falls; Program Chairman, Betty Crow; Faculty Sponsor, Mamie Braswell; Corresponding Secretary, Rosa Lea Jackson.

Kappa Mu Epsilon, Central College (Fayette, Missouri)

The *Missouri Epsilon* Chapter of *Kappa Mu Epsilon* at Central College enjoyed the following papers during 1949–50:

Newton and Leibniz, by Paul Calvert

Hamilton and quaternions, by Mark Barton

The kinetic theory of gasses, by Merle Cartwright

An introduction to modern geometry, by Dr. Floyd Helton

Cantor's theory of infinity, by Niels Nielson

A brief history of the development of the engineering profession, by Prof. C. Denny

History of the number system, by David Bouldin

Gauss, Prince of Mathematics, by Norman Drissell

Galileo, the Star Gazer, by Eva Gilbertson

The duodecimal system of counting, by J. P. Karnes

Methods of summability of infinite series, by Dr. Paul Burcham, Chmn. of Department of Mathematics, University of Missouri

Evariste Galois, by Mary J. Groce

Calculating prodigies, by Robert Christian

Peaucellier's cell, by Mary Cronbaugh.

At the final meeting of the year, a picnic dinner for members and guests was held at the home of the Sponsor, Dr. Floyd Helton.

NEWS AND NOTICES

EDITED BY EDITH R. SCHNECKENBURGER, University of Buffalo

Readers are invited to contribute to the general interest of this department by sending news items to Edith R. Schneckenburger, University of Buffalo, Buffalo 14, New York. Items must be submitted at least two months before publication can take place.

THE ELEVENTH ANNUAL WILLIAM LOWELL PUTNAM MATHEMATICAL COMPETITION

The eleventh annual William Lowell Putnam Mathematical Competition will be held on Saturday, March 31, 1951. This competition, made possible by the trustees of the William Lowell Putnam Intercollegiate Memorial Fund left by Mrs. Putnam in memory of her husband, is under the sponsorship of the Mathematical Association of America and is open to undergraduate students in universities and colleges of the United States and Canada who have not received

a degree. The examination will consist of two parts of three hours each. The questions will be taken from the fields of calculus (elementary and advanced) with applications to geometry and mechanics not involving techniques beyond the usual applications, higher algebra (determinants and theory of equations), elementary differential equations, and geometry (advanced plane and solid analytic geometry). Any college or university wishing to enter a team or individual contestants may secure an application blank from Professor L. E. Bush 112 Albertus Magnus Hall, College of St. Thomas, St. Paul 1, Minnesota, by a postcard request. All applications must be filed with the Director not later than March 1, 1951. If three candidates are presented from a college or university, they are to constitute a team; if more than three are presented from any one college or university, the team of three must be named on the application. Fewer than three from one college or university may compete as individuals.

The examination may be given at any place where a team, or at least three candidates, can be assembled. Exceptions to this rule may be made by the Director in cases where it would entail unusual inconvenience to a contestant. Sealed copies of the examinations will be sent to the supervisor of the examination in time for the examination day and are not to be opened before the hour set.

The prizes to be awarded to the departments of mathematics of the institutions with the winning teams are $400, $300, $200 and $100, in the order of their rank. In addition, there will be prizes of $40, $30, $20 and $10 awarded to the members of these teams according to the rank of the team; a prize of $50 to each of the five highest contestants and a prize of $20 to each of the succeeding five highest contestants. Each of the winners will receive a suitable medal. Honorable mention will be given to several teams next in order after the four winning teams and to several individuals next in order after the ten individual winners. For further encouragement of the Competition, there will be awarded at Harvard University (or at Radcliffe College in the case of a woman) an annual $1800 William Lowell Putnam Prize Scholarship to one of the first five contestants, this to be available either immediately or on the completion of the student's undergraduate work.

Reports on the ten previous competitions and examination questions will be found in this MONTHLY for May, 1938, 1939, 1940, 1941, 1942, October, 1946, August-September, 1947, December, 1948, and August-September, 1949, 1950.

COMMITTEE ON THE MATHEMATICAL TRAINING OF SOCIAL SCIENTISTS

At the summer meeting of the Association at the University of Colorado on August 29–30, 1949 there was held a Symposium on the Mathematical Training of Social Scientists (this MONTHLY, vol. 56, 1949, p. 658). The Symposium was sponsored by the Mathematical Association of America, the Institute of Mathematical Statistics and the Econometric Society. A summary of the statements made by the participants in the Symposium has been printed in Econometrica, vol. 18, 1950, pp. 193–205.

At the conclusion of the Symposium it was voted that the officers of the participating societies be requested to appoint a joint committee to study the need for better mathematical training of social scientists and the ways and means of improving their mathematical preparation.

In accordance with this resolution a Committee on the Mathematical Training of Social Scientists has been established under the chairmanship of Professor W. G. Madow of the University of Illinois. Twelve organizations interested in the problem are represented on the committee. The representatives of the Mathematical Association of America are Professors F. L. Griffin and E. P. Northrop. Members interested in the work of this committee are urged to communicate with our representatives on the committee.

MATHEMATICAL COLLOQUIUM AT THE UNIVERSITY OF ST. ANDREWS

A Mathematical Colloquium under the auspices of the Edinburgh Mathematical Society will be held at the University of St. Andrews on July 18–28, 1951. The Colloquium will consist of three or four short courses of lectures on topics of general interest, supplemented by several individual lectures on more specialized subjects. Professor H. S. M. Coxeter of the University of Toronto and Professor J. L. Synge of Dublin Institute for Advanced Studies will give courses. Other courses are being arranged. As accommodation in the University residence halls may be limited, application for membership should be made as soon as possible to the Colloquium Secretary, Dr. D. E. Rutherford, United College, St. Andrews, Scotland. Further information can be obtained from the Secretary.

THE MATHEMATICAL ASSOCIATION OF AMERICA

Official Reports and Communications

APRIL MEETING OF THE SOUTHEASTERN SECTION

The annual meeting of the Southeastern Section of the Mathematical Association of America was held at the University of Florida, Gainesville, Florida, on Friday and Saturday, April 7–8, 1950. Professor F. A. Lewis, Chairman of the Section, presided at the Friday afternoon and Saturday morning meetings of the subsections, which were presided over by Professors C. G. Phipps, George Sauté, R. J. Levit, T. M. Simpson, F. W. Kokomoor, C. L. Seebeck, Jr., L. A. Dye, and W. A. Gager. Professor C. G. Phipps presided on Friday evening at the dinner.

There were about two hundred and fifty present, including the following one hundred and eleven members of the Association: W. H. Badgley, Jr., W. S. Beckwith, R. G. Blake, F. C. Bolser, Floyd Bowling, A. T. Brauer, N. R.

Bryan, R. K. Butz, T. C. Carson, J. D. Charron, C. E. Clark, M. J. Cleveland, Reginal Cobb, A. C. Cohen, Jr., J. B. Coleman, R. W. Cowan, R. R. Croxton, J. C. Currie, R. D. Depew, N. E. Dodson, R. D. Doner, L. A. Dye, J. C. Eaves, G. B. Findley, R. B. Folsom, Tomlinson Fort, W. A. Gager, D. B. Goodner, S. T. Gormsen, E. H. Hadlock, B. F. Hadnot, G. B. Huff, W. R. Hutcherson, Rosa L. Jackson, A. J. Killebrew, C. E. Kirkwood, Jr., F. W. Kokomoor, J. W. LaGrone, G. B. Lang, J. W. Lasley, Jr., R. J. Levit, F. A. Lewis, M. E. McCarty, W. G. McGavock, S. W. McInnis, J. D. Mancil, Nathaniel Macon, C. F. Martin, J. E. Martin, R. H. Mason, W. N. Mebane, Jr., H. A. Meyer, Herman Meyer, E. P. Miles, Jr., W. L. Miser, J. A. Morales, J. C. Morelock, H. W. Morrow, Jr., K. W. Morrow, W. B. Moye, T. F. Mulcrone, T. A. Newton, A. J. Owens, W. V. Parker, P. B. Patterson, W. D. Peeples, Jr., Lillian G. Perkins, P. B. Peyton, Jr., C. G. Phipps, Z. M. Pirenian, P. A. Piza, M. D. Prince, F. M. Rankin, Ellen F. Rasor, B. P. Reinsch, G. E. Reves, T. L. Reynolds, M. C. Rhodes, J. M. Robertson, A. J. Robinson, H. A. Robinson, L. V. Robinson, George Sauté, P. C. Scott, C. L. Seebeck, Jr., E. B. Shanks, D. C. Sheldon, T. M. Simpson, C. B. Smith, W. R. Smythe, Jr., W. S. Snyder, W. H. Spragens, Jr., F. W. Stallard, E. L. Stanley, P. M. Swingle, J. M. Thomas, S. L. Thompson, H. S. Thurston, E. A. Voorhees, Jr., B. T. Wade, T. L. Wade, Jr., D. T. Walker, F. A. Wallace, Betty R. Weber, W. W. Weber, Ernest Williams, W. H. Wilson, Herbert Wolf, G. N. Wollan, J. W. Young, R. W. Young.

At the business session the following officers were elected: Chairman, C. G. Phipps, University of Florida; Vice-Chairman, B. G. Clark, Vanderbilt University; Secretary-Treasurer, H. A. Robinson, Agnes Scott College. The Section voted to hold its next meeting on March 16–17, 1951, at Vanderbilt University and George Peabody College.

The program consisted of the following papers:

1. *A note on problem number 4288 in the American Mathematical Monthly, January 1950*, by Professor C. G. Phipps, University of Florida.

This problem was proposed in the March 1948 issue of this MONTHLY. Two purported solutions were published in the January 1950 issue. These solutions appear to contain several misinterpretations. The present note explained the derivation of the probabilities of the events to which the probability 13/41 applies. The analysis was based on a corrected diagram.

2. *Partial fractions*, by Professor J. M. Thomas, Duke University.

It was shown that synthetic division can be used to develop the theory completely and to calculate the coefficients systematically.

3. *Simultaneous linear equations*, by Professor L. A. Dye, The Citadel.

This paper presents an orderly, economical method of solving any system of simultaneous linear equations possessing a solution. A check on the computations is provided at each step and inconsistent equations are easily recognized as such.

4. *A criterion for a common root of k algebraic equations*, by Professor A. T. Brauer, University of North Carolina.

This paper has appeared in this MONTHLY for May, 1950.

5. *Graph paper as a teaching aid*, by Professor Floyd Bowling, Lincoln Memorial University.

The author demonstrated two types of graph paper which he designed, one for trigonometric curves, and the other for three dimensions. The latter is useful in solid geometry, solid analytics and some phases of engineering drawing.

6. *The linear congruence group modulo n*, by Professor F. A. Lewis, University of Alabama.

The speaker discussed the results of a study of $GLH(m, n)$, the group of linear homogeneous transformations on m variables whose coefficients range over the integers $0, 1, \cdots, n-1$ in such a way that the determinant of each transformation is relatively prime to n. Properties of $GLH(m, n)$ were developed by the general linear homogeneous group on m variables with coefficients in the $GF[p^n]$.

7. *The brachistochrone problem*, by W. R. Smythe, Jr., Rollins College.

In the first part of this paper, the path of minimum time of descent between two fixed points was discussed. In the second part, the path from a point to a curve, including Barnett's construction for the focal point, was given.

8. *Mathematics in art and nature*, by Sarah I. Hancock, Agnes Scott College, introduced by the Secretary.

In this paper, the author discussed certain curves, numbers, and geometric forms which appear in art and nature.

9. *Notes on certain unsolved problems*, by E. A. Voorhees, Jr., Vanderbilt University.

In this paper the explanation, progress, and by-products of the investigation of certain of the more elementary unsolved problems, including some of the so-called "Hilbert problems," were discussed.

10. *A prime-representing function*, by D. T. Walker, University of Georgia.

A theorem given by W. H. Mills in June 1947 *Bulletin of the American Mathematical Society* was discussed and extended. Between the cube of an integer and that of the next higher integer less one there is always at least one prime. From this a bounded monotone increasing sequence of real numbers was constructed.

11. *Some theorems in matrix algebra*, by Professor T. L. Wade, Florida State University.

With the use of the delta and epsilon permutation symbols (numerical tensors) and identities on them, explicit proofs are given of the theorem "if A is a matrix with n columns and B is a matrix with n rows, then any m-rowed determinant of the matrix $C=AB$ is equal to the sum of terms, each of which is the product of an m-rowed determinant of A and an m-rowed determinant of B," and related theorems.

12. *A generalization of the field of values of a matrix*, by Professor J. C. Currie, Georgia Institute of Technology.

For a given matrix A, and a positive definite Hermitian matrix S, the set of complex numbers $xAS\bar{x}'$, where x ranges over the set of vectors such that $x\,S\,\bar{x}'=1$, is called the S-field of values

of A. The nature of this S-field, its relation to the ordinary field of values (for $S=I$), and some results of its application to the problem of limits for the characteristic roots of A were discussed.

13. *A method of construction of a properly primitive positive ternary quadratic form*, by Professor E. H. Hadlock, University of Florida.

The author gave an example illustrating a method of construction of a properly primitive positive ternary quadratic form when the invariants r and delta and the associated progression were known.

14. *A new interpolation formula*, by Professor C. L. Seebeck, Jr., University of Alabama.

The formula involves m derivatives evaluated at $n+1$ points, and reduces to Lagrange's formula when $m=0$. It is

$$f(x) = \sum_{k=0}^{n} \Big[\prod_{k \neq i} (x - x_i) S_k \Big/ \prod_{k \neq i} (x_k - x_i) \Big] + R,$$

$$S_k = \sum_{j=0}^{m} \frac{(m+n-j)!}{(m+n)!} {}_mC_j f^{[j]}(x_k).$$

15. *An algebraic geometry formula*, by J. C. Morelock, University of Florida.

The formula referred to may be used for writing the terms of any homogeneous polynomial of prime degree in four variables. The method permits a classification of the terms of the polynomial into sets where each set is invariant under the same special linear transformation. The method also affords a new way to count the number of terms in the most general polynomial of prime degree.

16. *Geometric aspects of the euclidean algorithm in imaginary quadratic domains*, by Professor H. S. Thurston, University of Alabama.

Professor Thurston demonstrated with diagrams the existence or non-existence of the euclidean algorithm in quadratic domains $R_a[\sqrt{m}]$, where $m<0$.

17. *Plane analytic geometry of the complex domain*, by B. F. Hadnot, University of Georgia.

By use of a one to one correspondence between the field of complex numbers and points oriented by the rectangular coordinate system, Mr. Hadnot outlined a study of straight lines and conics with as little reference as possible to the plane analytics of the real domain.

18. *Effect of a rigid elliptic disk on the stress distribution in an orthotropic plate*, by A. J. Owens, University of Florida.

An analysis of the stress distribution in an orthotropic plate was made from a study of a stress function of two complex variables. The results were applied to a Sitka spruce plate in a state of plane stress.

19. *The Monte Carlo method*, by Professor W. S. Snyder, University of Tennessee.

The Monte Carlo method of solving a problem consists in devising a mathematical model involving a stochastic variable whose expected value is the answer to the problem, and in evaluating the expected value by sampling. In many cases the mathematical model is at once suggested by the problem, but the method is proving adaptable to such problems as the inversion of matrices, solution of integral equations, evaluation of integrals, and determination of eigenvalues and eigenfunctions of certain partial differential equations. The difficulties of sampling are often less serious than

those encountered in attempting a solution by the customary algebraic or analytic procedures. Present experience seems too limited to characterize the class of problems where the method may be used, or to give very precise indications of its relative efficiency.

20. *A grade crossing quadratic*, by Doctor J. B. Coleman, formerly with the South Carolina Highway Department.

Where visibility is limited at a grade crossing the safe speed of approach will depend upon the distance at which visibility occurs, the time reaction of the driver, and the rate of deceleration. Finding the safe speed involves the solution of a quadratic whose coefficients have physical significance, the importance of which may not be realized by the average driver.

21. *A simple perfect point becomes a triple point*, by Professor W. R. Hutcherson, University of Florida.

Curves of order 21 determined by cubic and seventh degree surfaces were studied. These curves were invariant under a certain transformation. A simple perfect invariant point on the seventh degree surface became a triple point on the cubic curve in the tangent plane.

22. *Applications of summation by parts in the theory of summability of infinite series*, by Professor Tomlinson Fort, University of Georgia.

In this paper the author established certain tests for the summability of series by a triangular matrix analogous to well-known tests for convergence.

23. *Some remarks on the Monte Carlo method*, by Professor A. C. Cohen, Jr., University of Georgia.

A discussion of the statistical approach in the numerical evaluation of n-fold integrals was given. When n is large, for example when $n=10$, ordinary methods of numerical integration require an abnormal length of time to complete even when modern high-speed electronic computing machines are employed. By use of the Monte Carlo method, such computations can be quickly completed with sufficient accuracy for most practical purposes.

24. *Some plane curves drawn by a method suggested by a common construction of an ellipse*, by Professor W. L. Miser, Vanderbilt University.

The common method of constructing an ellipse from two concentric circles and the line $y=mx$ suggested the problem of constructing a curve which comes from replacing the two circles by two general conics. An equation of degree eight in x and y is found. Some simple examples are given, among them being the witch, the pseudo-witch, the serpentine, and other simple curves.

25. *Continuity concepts in college algebra*, by Professor J. D. Mancill, University of Alabama.

The need for the treatment of the concept of continuity of the real number system in college algebra is stressed, particularly as a prerequisite for calculus. Methods and means of presenting concepts of continuity in college algebra are demonstrated.

26. *On the diophantine problems that arise in teaching*, by Professor G. B. Huff, University of Georgia.

Most teachers of elementary mathematics are familiar with the fact that diophantine problems occur in constructing "nice" numerical problems. It was shown that for many of these problems, satisfactory solutions may be obtained by reducing the problem to one that can be solved by factoring a determinate integer.

27. *Electronic differentiating and integrating circuits and their use as teaching aids*, by M. D. Prince, Georgia Institute of Technology.

This speaker described the operation and use of an electronic device which performs differentiation and integration of periodic functions. The technique employed permits direct observation of the input and output functions by means of a standard cathode-ray oscilloscope which traces out the curves on its screen.

28. *Newton and his Principia Mathematica*, by Professor W. A. Gager, University of Florida.

In this descriptive paper attention was called to certain highlights in the *Principia* such as in Book I, laws of motion and fluxion; in II, motion of bodies in resisting media; and in III, Newton's "system of the world." Behind a great book there must exist a great personality. Many interesting phases of Newton's life were discussed.

29. *The use of group diagrams in the theory of finite groups*, by W. D. Peeples, Jr., University of Georgia.

The author considered the construction of group diagrams when the generators and defining relations are known, and he illustrated same by solving a particular problem.

30. *A study of a certain transformation in the complex plane*, by Polly A. Harris, Agnes Scott College, introduced by the Secretary.

This paper consisted of a study of the properties of the function $w=z/(z^3-1)$ when the function is considered as a transformation between the z and w planes.

31. *Kummer's test for convergence obtained by the Abel-Dini theorem*, by T. A. Newton, University of Georgia.

The Abel-Dini theorem and the Pringsheim modification were used to obtain a generalization of the comparison test of the second kind for infinite series. The result was Kummer's test for convergence.

32. *The evaluation of certain definite integrals and some applications to heat flow*, by M. D. Findley, University of Florida, introduced by the Secretary.

By contour integration in the complex plane, integrals of the type

$$\int_0^\infty \sinh ax \cos bx(\sinh mx \cosh nx)^{-1}dx$$

were evaluated. An application of the integral was given where the temperature distribution along one edge of an infinite strip was represented by B sech kx and the other edge was kept at zero.

33. *A proof of a theorem from Bromwich's "Infinite Series,"* by G. N. Wollan, North Georgia College.

A theorem stating a necessary and sufficient condition under which alternating series of a certain type can be multiplied by the Cauchy rule was proved directly from the definition of convergence.

34. *Iterated sums of powers of the binomial coefficients*, by Professor E. B. Shanks, Vanderbilt University.

For arbitrary positive integers i, k, n, p, $(i \leqq n)$, let

$$s_i^k(n, p) = \sum_{j=1}^{n} s_i^k(j, p-1) \quad \text{where} \quad s_i^k(n, 0) = \binom{n}{i}^k.$$

It is shown that

$$s_i^k(n, p) = \sum_{j=1}^{ik-i+1} A_j \binom{r+ik-i+p-j+1}{ik+p}$$

where $A_1=1$ and $A_j=A_{ik-j+2}$ for $j=1, 2, \cdots, ik-i+1$. The A_j are positive integers independent of n and p, whose values are given by a determinant of order j. This result is a two parameter generalization of Bernoulli's "Summa Potestatum."

35. *A mechanical system related to a continued fraction*, by Leo Moser, University of North Carolina, by title.

The oscillations of a certain mechanical system were shown to have a close connection with the continued fraction expansion of a certain quadratic surd. The possibility of obtaining a mechanical system similar in structure to a given continued fraction was discussed.

36. *On series of Walsh functions*, by Professor W. H. Spragens, Jr., Florida State University.

Professor Spragens extended a result of Walsh on series of certain eigenfunctions which he proved to be equiconvergent with Fourier sine series for functions in L^2. It was shown that this result may be extended to functions in L. Extensions to double series were indicated.

37. *A generalization of the remainder theorem*, by Professor R. J. Levit, University of Georgia.

The familiar theorem, "If a polynomial $p(x)$ irreducible over a field K has a root in common with a polynomial $f(x)$ over K, then $p(x)$ divides $f(x)$," is a generalization of the factor theorem. In this paper a corresponding generalization of the remainder theorem was discussed.

38. *Powers of triangular numbers*, by Mr. P. A. Piza, San Juan, Puerto Rico.

This paper, privately printed in English, will appear in French in *Mathesis*. The author presented many interesting properties of the powers of triangular numbers, Fermat coefficients, and tetrahedral numbers.

39. *A new generalization of Legendre's differential equation*, by Professor R. W. Cowan, University of Florida.

The differential equation is taken in a form containing two parameters, and is solved by the method of Frobenius. For certain particular values of the parameters the general solution reduces to polynomials. It is shown that the polynomials are orthogonal over the interval $(-1, 1)$ with respect to a certain weight function.

40. *A further generalization of the Leibnitz theorem*, by Professor L. V. Robinson, University of South Carolina.

When D in $F(D)$ of the Leibnitz theorem is replaced by X defined as the sum of terms g_kD_k, where g_k is a function of n independent variables x_i, and D_k is the partial differential operator of x_k, it is shown that a more general theorem is thus obtained.

41. *Calculus concepts in connection with the quadratic function*, by Professor N. R. Bryan, Clemson College.

Professor Bryan suggested how continuity may be maintained in the teaching of quadratic functions in algebra and in calculus.

42. *Extended integration by parts*, by B. F. Hadnot, University of Georgia.

By repeated integration by parts, Taylor's formula and certain indefinite integrals of a given type of determinant were obtained.

43. *On Akilov's Theorem*, by Professor D. B. Goodner, Florida State University.

Akilov (*Doklady Akad. Nauk SSSR* (N.S.) 57(1947), 643–46) proved the theorem by using Kantorovich's extension theorem. This speaker presented a direct proof of Akilov's theorem and certain ramifications of the theorem. Certain spaces having property P were characterized.

44. *Discussion of classroom notes and junior college mathematics.*

This discussion was led by Professor W. A. Gager, University of Florida.

H. A. ROBINSON, *Secretary*

CALENDAR OF FUTURE MEETINGS

Thirty-fourth Annual Meeting, University of Florida, Gainesville, December 30, 1950.

Joint Meeting with American Society for Engineering Education, Michigan State College, East Lansing, June 25–26, 1951.

Thirty-second Summer Meeting, University of Minnesota, Minneapolis, September 3–4, 1951.

The following is a list of the Sections of the Association with dates of future meetings so far as they have been reported to the Secretary.

ALLEGHANY MOUNTAIN, Duquesne University, Pittsburgh, Pennsylvania, May, 1951.

ILLINOIS, University of Illinois, Urbana, May 11–12, 1951.

INDIANA, May 5, 1951.

IOWA, Wartburg College, Waverly, April 20–21, 1951.

KANSAS

KENTUCKY, Eastern Kentucky State College, Richmond, April 28, 1951.

LOUISIANA-MISSISSIPPI, Mississippi State College, State College, February 16–17, 1951.

MARYLAND-DISTRICT OF COLUMBIA-VIRGINIA, Catholic University, Washington, D. C., December 9, 1950.

METROPOLITAN NEW YORK, Spring, 1951.

MICHIGAN, Michigan State College, East Lansing, March 24, 1951.

MINNESOTA, College of St. Benedict, St. Joseph, April 28, 1951.

MISSOURI, Central College, Fayette, Spring, 1951.

NEBRASKA, University of Nebraska, Lincoln, May 5, 1951.

NORTHERN CALIFORNIA, University of San Francisco, January 27, 1951.

OHIO, Columbus, April 21, 1951.

OKLAHOMA

PACIFIC NORTHWEST, State College of Washington, Pullman, June 15, 1951.

PHILADELPHIA

ROCKY MOUNTAIN, Colorado State College of Education, Greeley, April, 1951.

SOUTHEASTERN, Vanderbilt University and Peabody College, Nashville, Tennessee, March 16–17, 1951.

SOUTHERN CALIFORNIA, Whittier College, Whittier, March 10, 1951.

SOUTHWESTERN, University of New Mexico, Albuquerque, Spring, 1951.

TEXAS, Southern Methodist University, Dallas, Spring, 1951.

UPPER NEW YORK STATE, Hamilton College, Clinton, May 5, 1951.

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Edited by HOWARD EVES, Oregon State College, and
E. P. STARKE, Rutgers University

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